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DET
Department of Electronics and Telecommunications

Experimental Demonstration of a A Novel Update Algorithm in Stokes Space for Adaptive Equalization in Coherent Receivers

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Outline

- Stokes space representation
- Adaptive equalizer
  - Update rule in Stokes space
  - Optimum decision rule in Stokes space
- Simulation results
  - Comparison with CMA
- Experimental results
  - Comparison with CMA
- Conclusions

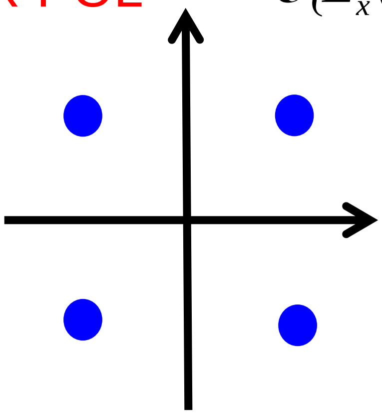
Stokes space representation - PM-QPSK

$$S_1(n) = |E_x(n)|^2 - |E_y(n)|^2$$

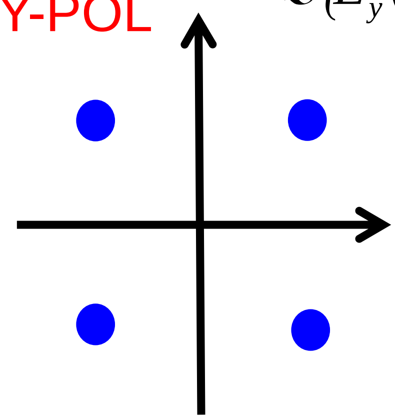
$$S_2(n) = 2\Re\{E_x(n)E_y(n)\}$$

$$S_2(n) = 2\Im\{E_x(n)E_y^*(n)\}$$

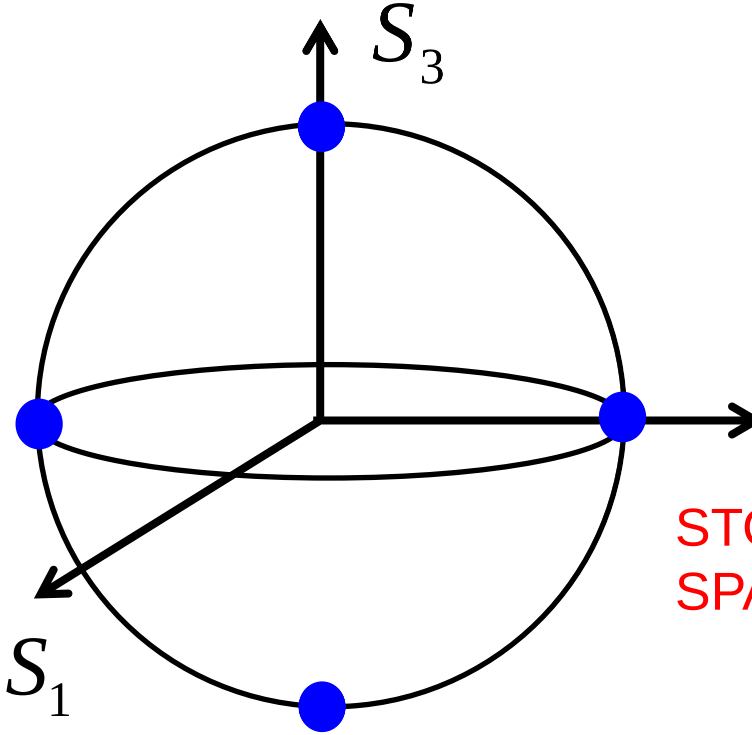
X-POL



Y-POL



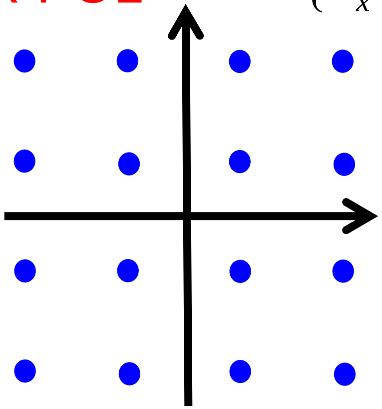
JONES SPACE



STOKES SPACE

Stokes space representation - PM-16QAM

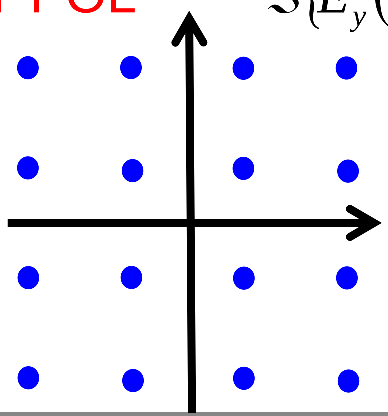
X-POL $\Im\{E_x(n)\}$



$\Re\{E_x(n)\}$

Y-POL $\Im\{E_y(n)\}$

JONES SPACE

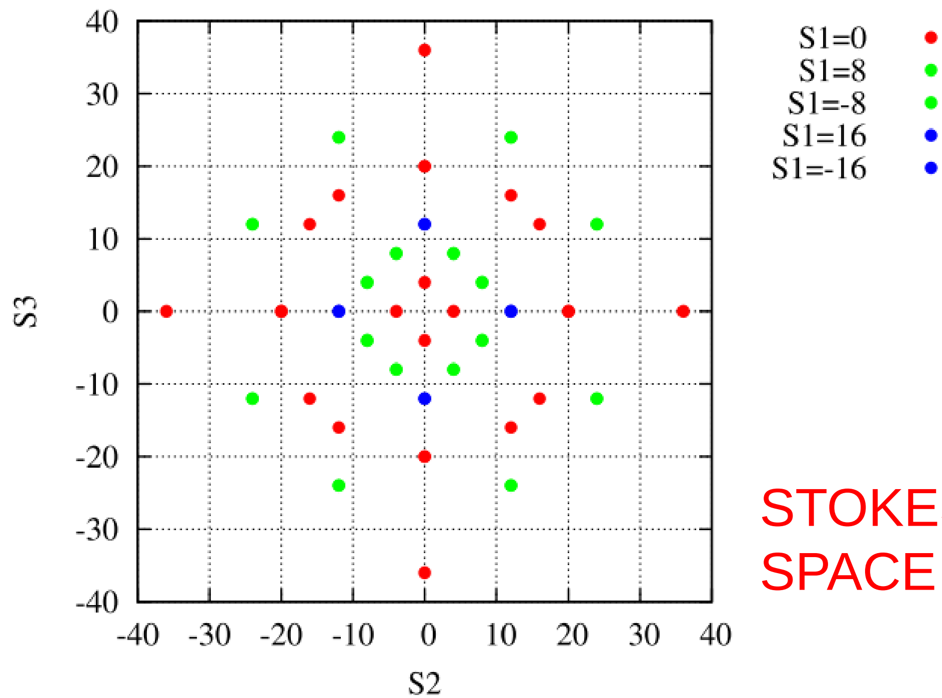


$\Re\{E_y(n)\}$

$$S_1(n) = |E_x(n)|^2 - |E_y(n)|^2$$

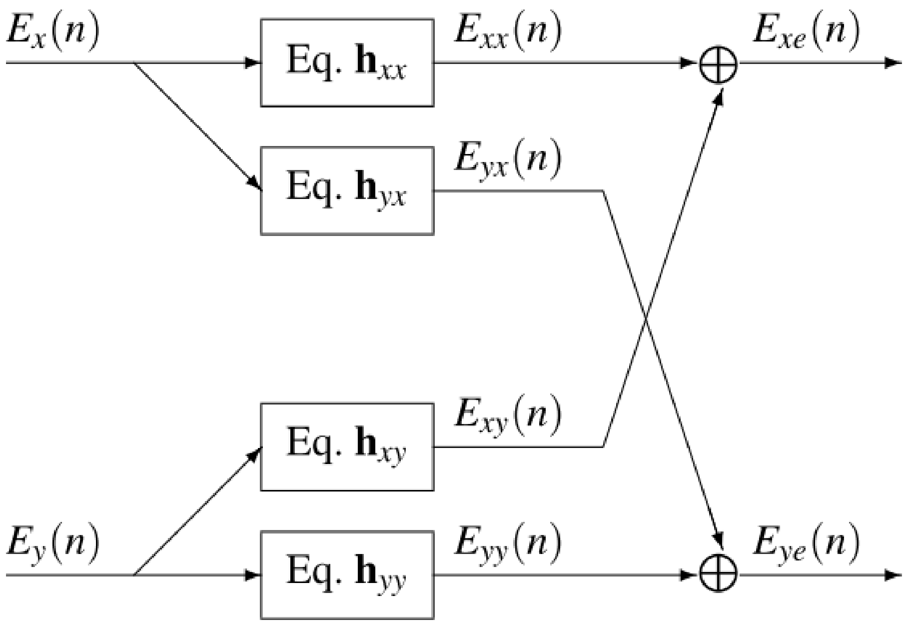
$$S_2(n) = 2\Re\{E_x(n)E_y(n)\}$$

$$S_2(n) = 2\Im\{E_x(n)E_y^*(n)\}$$



STOKES SPACE

Adaptive equalizer



$$E_{xe}(n) = \mathbf{E}_x^T \mathbf{h}_{xx} + \mathbf{E}_y^T \mathbf{h}_{xy}$$
$$E_{ye}(n) = \mathbf{E}_x^T \mathbf{h}_{yx} + \mathbf{E}_y^T \mathbf{h}_{yy}$$

- Standard CMA or LMS algorithms: update of the coefficients based on error signals evaluated on the two-dimensional constellations (separate for the two polarizations)

- **New algorithm: update of the coefficients based on error signal evaluated in the Stokes space [1]**

[1] G. Bosco et al., OFC 2014, paper Th3E.6

Stokes space update rule

- Error function to be minimized

$$f(\mathbf{h}) = f(\mathbf{h}_{xx}, \mathbf{h}_{xy}, \mathbf{h}_{yx}, \mathbf{h}_{yy}) =$$
$$= \left(S_{1e}(n) - \hat{S}_1(n)\right)^2 + \left(S_{2e}(n) - \hat{S}_2(n)\right)^2 + \left(S_{3e}(n) - \hat{S}_3(n)\right)^2$$

  – with:

$$\mathbf{S}_e = \left[S_{1e}(n), S_{2e}(n), S_{3e}(n)\right]$$

Stokes vector of the equalized signal

$$\hat{\mathbf{S}} = \left[\hat{S}_1(n), \hat{S}_2(n), \hat{S}_3(n)\right]$$

Stokes vector of the transmitted signal

either known (training sequence) or estimated (decision-directed)

Taps update algorithm – Stokes/CMA

- Rule for adaptively update the equalizer weights:

$$\mathbf{h}_{xx}(n+1) = \mathbf{h}_{xx}(n) - \mu \nabla_{\mathbf{h}_{xx}} f(\mathbf{h}(n))$$
$$\mathbf{h}_{xy}(n+1) = \mathbf{h}_{xy}(n) - \mu \nabla_{\mathbf{h}_{xy}} f(\mathbf{h}(n))$$
$$\mathbf{h}_{yx}(n+1) = \mathbf{h}_{yx}(n) - \mu \nabla_{\mathbf{h}_{xy}} f(\mathbf{h}(n))$$
$$\mathbf{h}_{yy}(n+1) = \mathbf{h}_{yy}(n) - \mu \nabla_{\mathbf{h}_{yy}} f(\mathbf{h}(n))$$

- Evaluation of gradients:

$$\nabla_{\mathbf{h}_{xx}} f(\mathbf{h}(n)) = C_1(n)\mathbf{E}_x^*$$
$$\nabla_{\mathbf{h}_{xy}} f(\mathbf{h}(n)) = C_1(n)\mathbf{E}_y^*$$
$$\nabla_{\mathbf{h}_{yy}} f(\mathbf{h}(n)) = C_2(n)\mathbf{E}_y^*$$
$$\nabla_{\mathbf{h}_{yx}} f(\mathbf{h}(n)) = C_2(n)\mathbf{E}_x^*$$

Stokes algorithm

$$\begin{bmatrix} C_1(n) \\ C_2(n) \end{bmatrix} = \begin{bmatrix} \varepsilon_1(n) & \varepsilon_2(n) \\ \varepsilon_2^*(n) & -\varepsilon_1(n) \end{bmatrix} \begin{bmatrix} E_{xe}(n) \\ E_{ye}(n) \end{bmatrix}$$
$$\varepsilon_1(n) = S_{1e}(n) - \hat{S}_1(n)$$
$$\varepsilon_2(n) = \left(S_{2e}(n) - \hat{S}_2(n)\right) + j\left(S_{3e}(n) - \hat{S}_3(n)\right)$$

CMA algorithm

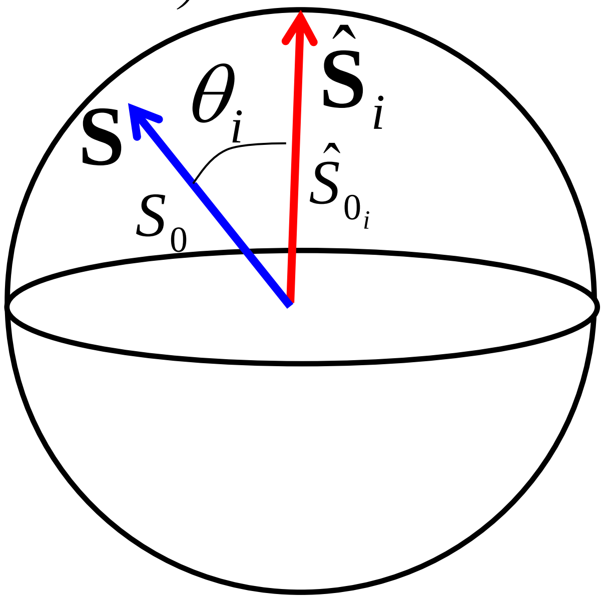
$$\begin{bmatrix} C_1(n) \\ C_2(n) \end{bmatrix} = \begin{bmatrix} \varepsilon_x(n) & 0 \\ 0 & \varepsilon_y(n) \end{bmatrix} \begin{bmatrix} E_{xe}(n) \\ E_{ye}(n) \end{bmatrix}$$
$$\varepsilon_x(n) = |E_{xe}(n)|^2 - R^2$$
$$\varepsilon_y(n) = |E_{ye}(n)|^2 - R^2$$

Statistics of noise in Stokes space

- $\mathbf{S} = (S_1, S_2, S_3)$ = noisy received vector
- $\widehat{\mathbf{S}}_i = (\hat{S}_{1_i}, \hat{S}_{2_i}, \hat{S}_{3_i})$ = ideal un-noisy constellation vector
- PDF of $\mathbf{S} | \widehat{\mathbf{S}}_i$ [*]:

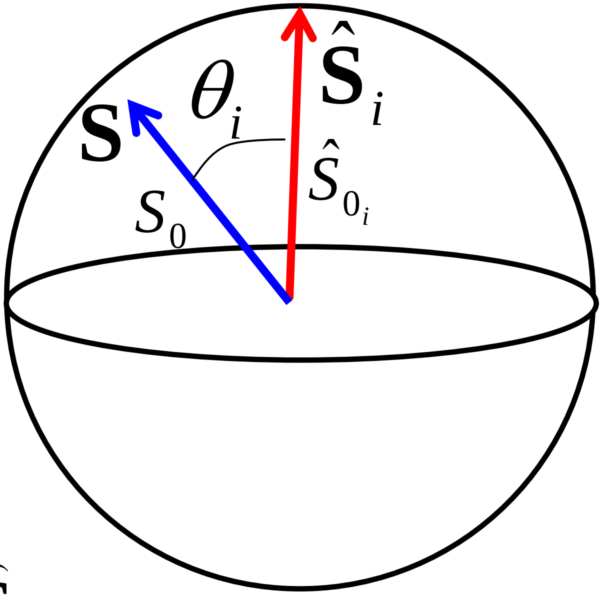
$$f_{\mathbf{s}|\widehat{\mathbf{s}}_i} = \frac{e^{-\frac{\hat{S}_{0_i} + S_0}{2\sigma^2}}}{16\pi S_0 \sigma^4} I_0\left(\frac{\sqrt{\hat{S}_{0_i} \cdot S_0}}{\sigma^2} \cos\left(\frac{\theta_i}{2}\right) \right)$$

  - S_0 = magnitude of $\mathbf{S}$
  - $\hat{S}_{0_i}$ = magnitude of $\widehat{\mathbf{S}}_i$
  - θ_i = angle between $\mathbf{S}$ and $\widehat{\mathbf{S}}_i$
  - σ^2 = noise variance in each polarization



Decision rule

$$f_{\mathbf{s}|\hat{\mathbf{s}}_i} = \frac{e^{-\frac{\hat{S}_{0_i}+S_0}{2\sigma^2}}}{16\pi\, S_0 \sigma^4} I_0\left(\frac{\sqrt{\hat{S}_{0_i}\cdot S_0}}{\sigma^2}\cos\left(\frac{\theta_i}{2}\right)\right)$$



- The decision rule can be based on the maximization of $f_{\mathbf{s}|\hat{\mathbf{s}}_i}$ over all possible noiseless constellation points $\hat{\mathbf{s}}_i$.
- There are common factors across all possible indices i that can be eliminated → we can apply the ML decision on the formula:

$$p_i = e^{-\frac{S_0}{2\sigma^2}} I_0\left(\frac{\sqrt{\hat{S}_{0_i}\cdot S_0}}{\sigma^2}\cos\left(\frac{\theta_i}{2}\right)\right)$$

Simplified metric in Stokes space

- Taking the logarithm and applying some simplifications, we obtain the following new metric (based on actual statistics in Stokes space):

$$m_i \approx -S_{0_i} + 2\sqrt{\hat{S}_{0_i}}\sqrt{S_0}\cos\left(\frac{\theta_i}{2}\right)$$

NOVEL METRIC

- Minimum-distance metric (based on Gaussian distribution hypothesis):

$$d_i^2 = \left(S_1 - \hat{S}_{1_i}\right)^2 + \left(S_2 - \hat{S}_{2_i}\right)^2 + \left(S_3 - \hat{S}_{3_i}\right)^2 = S_0^2 + \hat{S}_{0_i}^2 - 2\hat{S}_{0_i} S_0 \cos(\theta_i)$$

$$\Longrightarrow -\hat{S}_{0_i}^2 + 2\hat{S}_{0_i} S_0 \cos(\theta_i)$$

MINIMUM DISTANCE METRIC

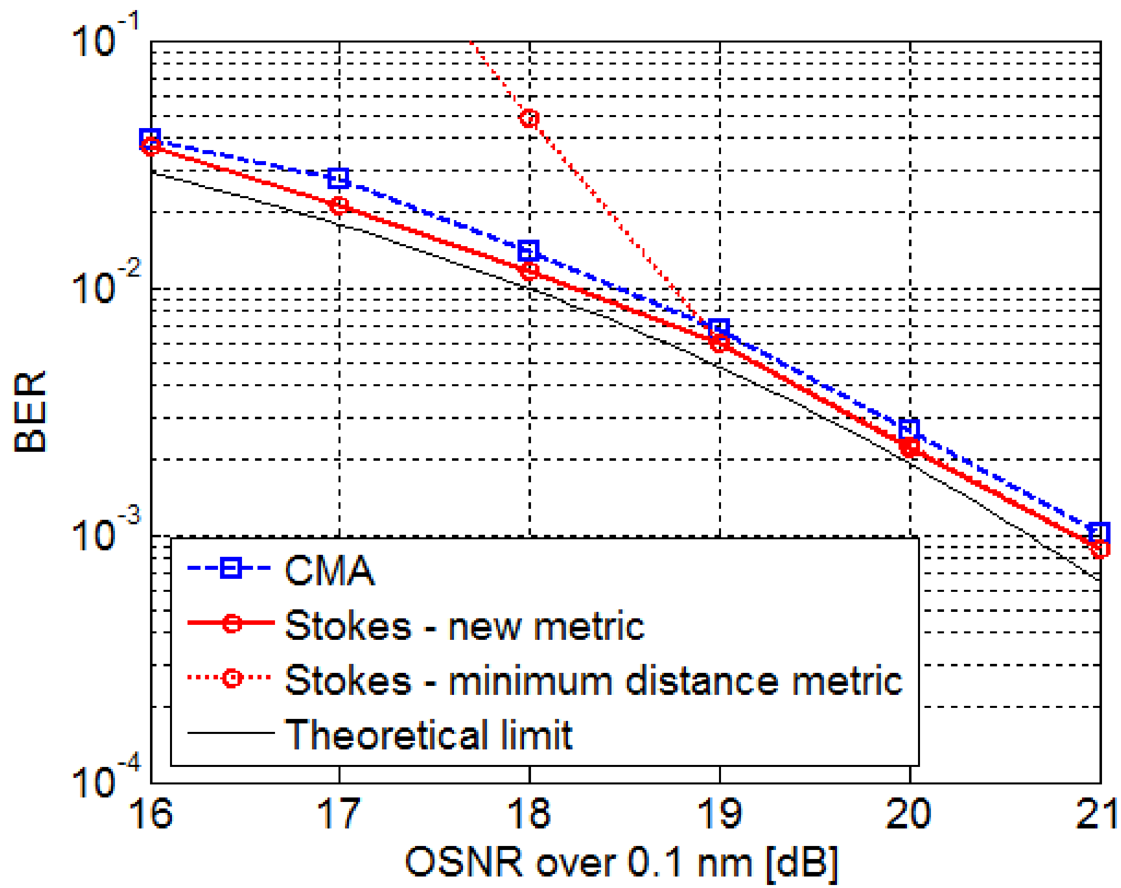
Case study – PM-16QAM

- Symbol rate: R_s=32 Gbaud
- Single-channel
- Nyquist spectrum (raised-cosine with roll-of 0.1)
- Residual CD = 250 ps/nm
- DGD = 1 symbol

- BER values estimated through Monte-Carlo simulation for several combinations of DGD axis and state of polarization (SOP) at the input of the Rx, for a total of ~900 cases

- Equalization using a training sequence, followed by decision-directed operation

BER vs. OSNR

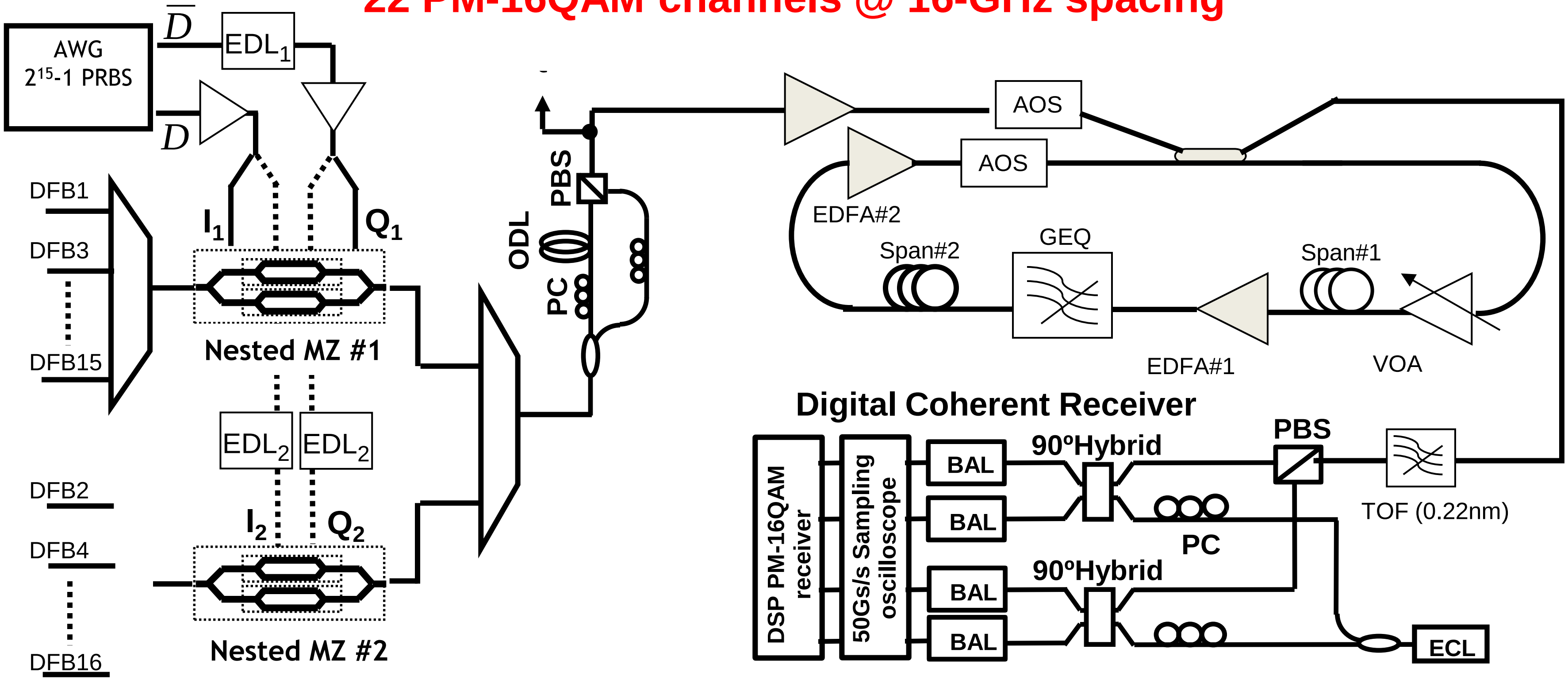


- Number of eq. taps: M = 31.
- The value of the adaptive equalizer update coefficient μ was optimized for both CMA and Stokes algorithms.

Experimental setup

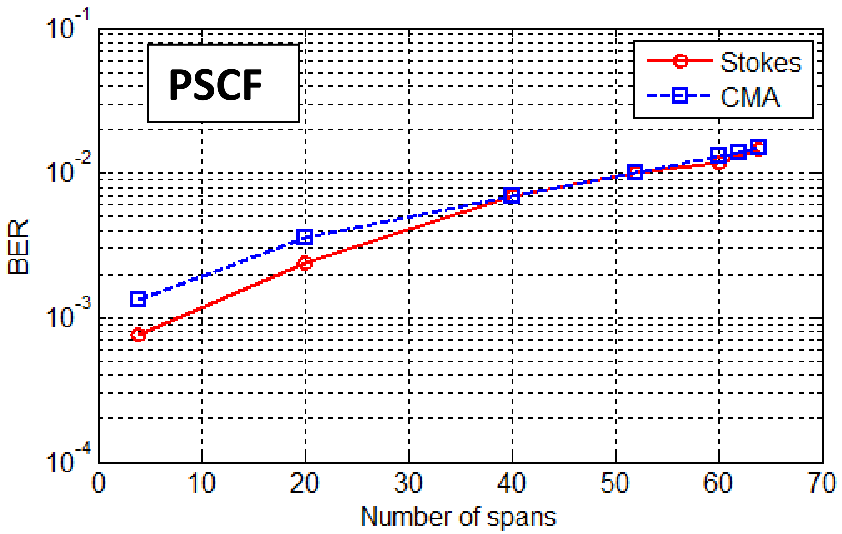
15.625 Gbaud

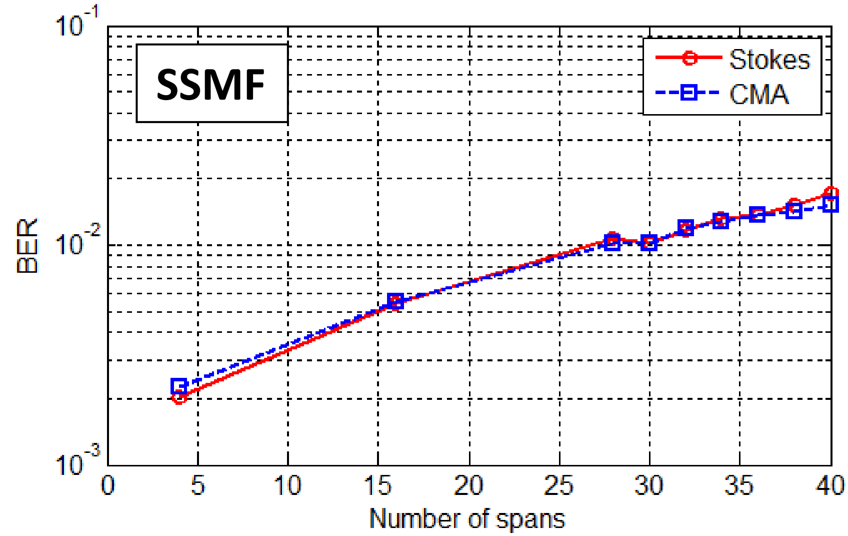
22 PM-16QAM channels @ 16-GHz spacing

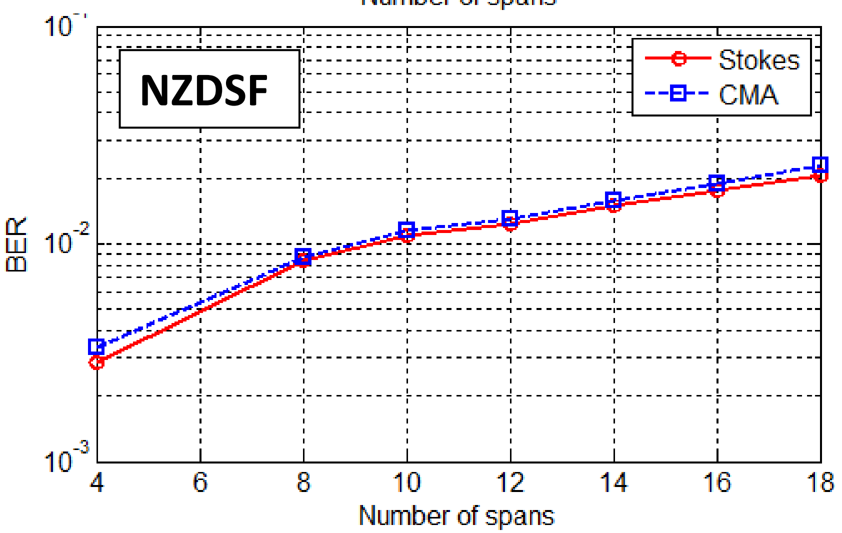


[2] A. Nespola et al., PTL, 26 (2), p. 296, 2014

Transmission results







Fiber parameters

Fiber	α [dB/km]	D [ps/ nm/km]	A_{eff} [mm^2]	L_{span} [km]
PSCF	0.162	20.92	131	54.42
SSMF	0.190	16.84	75	51.06
NZDSF	0.200	2.58	43	50.18

Conclusions

- We have described an adaptive equalizer update algorithm with the evaluation of error signals in Stokes space, introducing a novel decision metric, which outperforms the standard minimum distance metric.
- We have shown the first experimental demonstration of the use of the Stokes-space update algorithm in a high-spectral-efficiency long-haul transmission scenario, which confirm the preliminary performance assessment performed through numerical simulations

Thank you!

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