Analytical Results on Channel Capacity in Uncompensated Optical Links with Coherent Detection

Gabriella Bosco, Pierluigi Poggiolini, Andrea Carena, Vittorio Curri
Politecnico di Torino

Fabrizio Forghieri
Cisco Photonics Italy

ECOC 2011 - paper We.7.B.3

Outline

- Theory
  - Shannon formulas
  - Capacity of the polarization-multiplexed (PM) optical channel

- Results for uncompensated systems with EDFA amplification
  - Gaussian constellation
  - Realistic constellations with hard and soft decoding

Shannon formulas

- Capacity of the unconstrained AWGN channel:

$$C = \log_2(1+\mathrm{SNR}) \ \ [\mathrm{bits/symbol}]$$

- Capacity of the polarization-multiplexed (PM) optical channel:

$$C = 2\frac{R_s}{\Delta f}\log_2(1+\mathrm{SNR}) \ \ [\mathrm{bits/symbol}]$$

  - with

$$\mathrm{SNR} = \frac{B_n}{R_s}\mathrm{OSNR}$$

- R_s = symbol-rate
- Δf = frequency spacing between WDM channels
- B_n = reference noise bandwidth

Generalized OSNR

- According to the models presented in [1],[2], the system BER depends on a "generalized" OSNR:

$$\mathrm{OSNR}_{\mathrm{NL}} = \frac{P_{Tx,ch}}{P_{ASE} + P_{NLI}}$$

- $P_{Tx,ch}$ = signal power
- P_{ASE} = ASE noise power
- P_{NLI} = non-linear interference (NLI) power
- N_s = number of fiber spans
- F = EDFA noise figure
- α = fiber loss coefficient
- L_s = length of fiber span
- h = Plank's constant
- ν = center frequency

- In case of EDFA amplification:

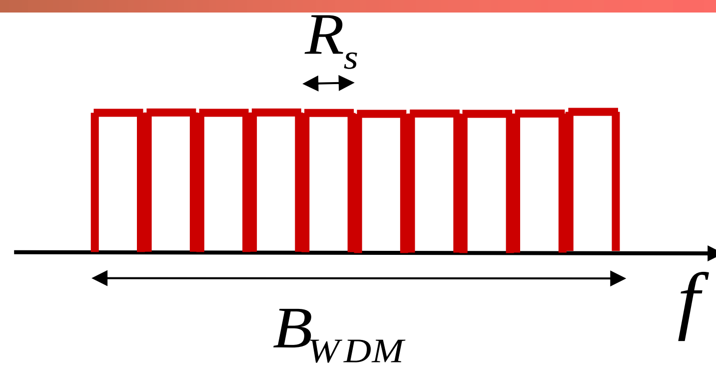
$$P_{ASE} = N_s F\left(e^{2\alpha L_s} - 1\right) h\nu B_n$$

[1] G. Bosco et al., "Performance Prediction for WDM PM-QPSK Transmission over Uncompensated Links", in Proc. of OFC 2011, paper OThO7, Mar. 2011.

[2] E. Grellier, A. Bononi, "Quality parameter for coherent transmissions with Gaussian-distributed nonlinear noise", Opt. Exp, **19**, pp.12781-12788 (2011)

NLI power at Nyquist limit

- At the Nyquist limit



the power of the non-linear interference can be analytically evaluated in uncompensated optical systems [3]:

$$P_{NLI} \approx \left(\frac{2}{3}\right)^3 N_s \gamma^2 L_{eff} G_{Tx}^3 \frac{\ln\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)}{\pi |\beta_2|} B_n$$

- β_2 = dispersion coefficient
- γ = non-linearity coeff.
- L_{eff} = fiber effective length

$$G_{Tx} = \frac{P_{Tx,ch}}{R_s} \qquad B_{WDM} = N_{ch} R_s \qquad L_{eff} = \frac{1 - e^{-2\alpha L_s}}{2\alpha}$$

[3] P. Poggiolini et al., "Analytical Modeling of Non-Linear Propagation in Uncompensated Optical Transmission Links", IEEE Photon. Technol. Lett. 23, 742-744 (2011).

Capacity of a PM optical channel

$$C = 2\log_2\left(1 + G_{Tx}\left[N_s\left(e^{2\alpha L_s} - 1\right)Fh\nu + \left(\frac{2}{3}\right)^3 \gamma^2 N_s L_{eff} G_{Tx}^3 \frac{\ln\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)}{\pi|\beta_2|}\right]^{-1}\right)$$

$$G_{Tx} = \frac{P_{Tx,ch}}{R_s} \qquad B_{WDM} = N_{ch} R_s$$

- N_s = number of spans
- L_s = length of fiber span
- β_2 = dispersion coefficient
- γ = non-linearity coeff.
- α = fiber loss coefficient
- L_{eff} = fiber effective length
- F = EDFA noise figure
- h = Plank's constant
- ν = center frequency
- R_s = symbol-rate

Capacity of a PM optical channel

$$C = 2\log_2\left(1 + G_{Tx}\left[N_s\left(e^{2\alpha L_s} - 1\right)Fh\nu + \left(\frac{2}{3}\right)^3 \gamma^2 N_s L_{eff} G_{Tx}^3 \frac{\ln\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)}{\pi|\beta_2|}\right]^{-1}\right)$$

$$G_{Tx} = \frac{P_{Tx,ch}}{R_s} \qquad B_{WDM} = N_{ch} R_s$$

- N_s = number of spans
- L_s = length of fiber span
- β_2 = dispersion coefficient
- γ = non-linearity coeff.
- α = fiber loss coefficient
- L_{eff} = fiber effective length
- F = EDFA noise figure
- h = Plank's constant
- ν = center frequency
- R_s = symbol-rate

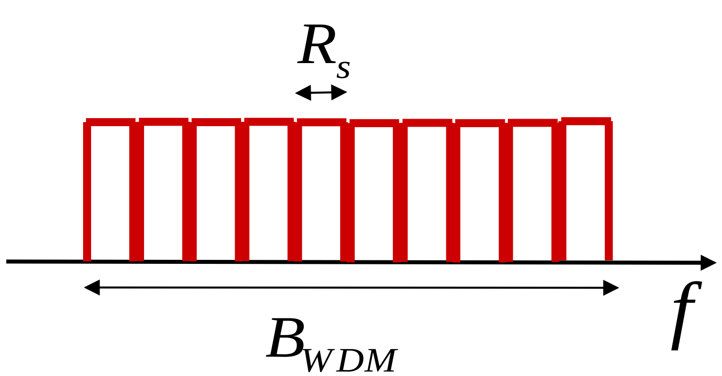
- Hypotheses:
  - EDFA amplification
  - Uncompensated transmission
  - Nyquist limit ($\Delta f = R_s$)
  - Ideal Gaussian constellation
  - Soft decision

Capacity of a PM optical channel

$$C = 2\log_2\left(1 + G_{Tx}\left[N_s\left(e^{2\alpha L_s} - 1\right)Fh\nu + \left(\frac{2}{3}\right)^3 \gamma^2 N_s L_{eff} G_{Tx}^3 \frac{\ln\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)}{\pi|\beta_2|}\right]^{-1}\right)$$

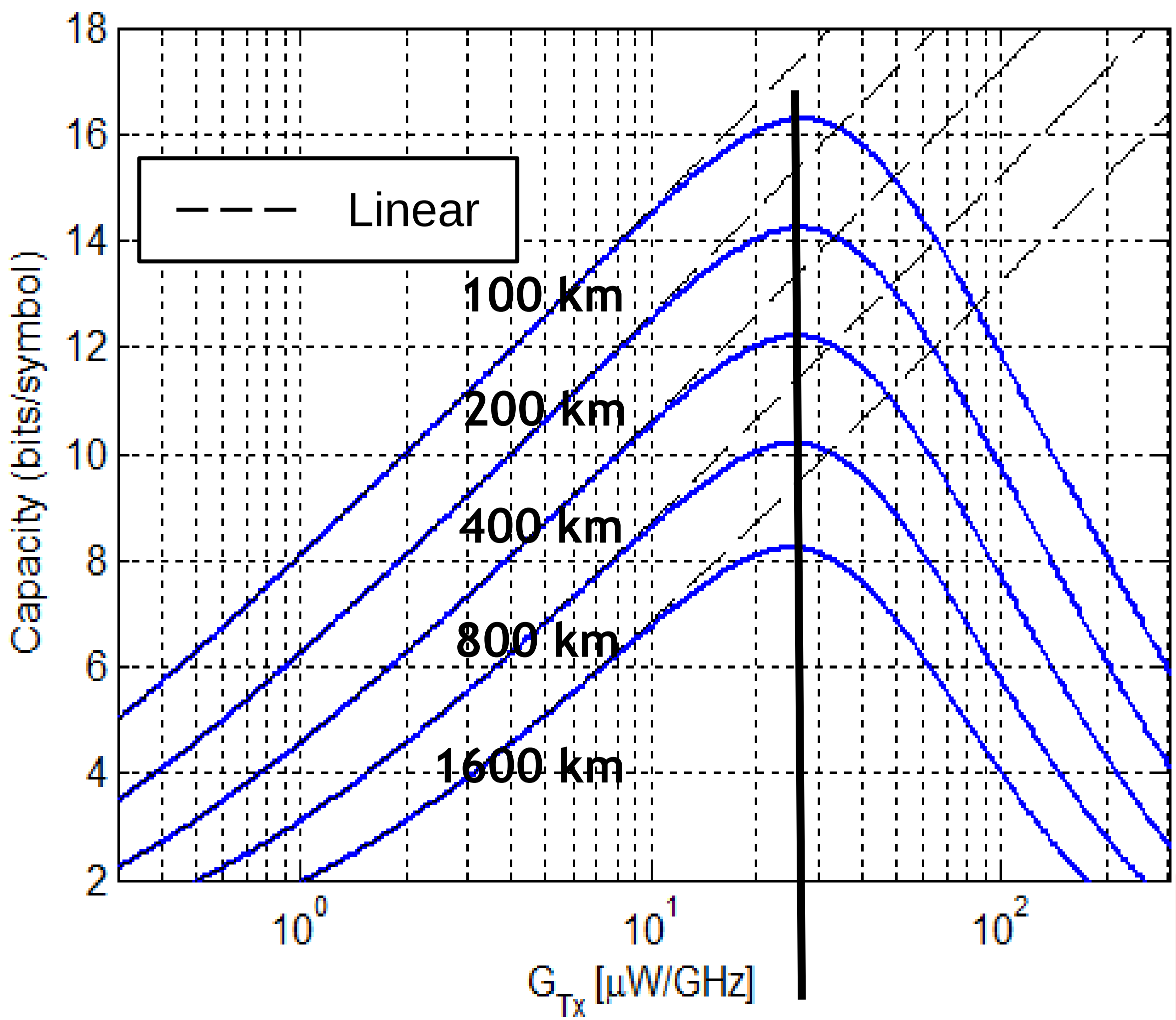
$$G_{Tx} = \frac{P_{Tx,ch}}{R_s} \qquad B_{WDM} = N_{ch} R_s$$

At the Nyquist limit, capacity is independent of the symbol-rate.



- N_s = number of spans
- L_s = length of fiber span
- β_2 = dispersion coefficient
- γ = non-linearity coeff.
- α = fiber loss coefficient
- L_{eff} = fiber effective length
- F = EDFA noise figure
- h = Plank’s constant
- ν = center frequency
- R_s = symbol-rate

Gaussian constellation



26.8 μW/GHz

- B_{WDM} = 4 THz (C-band)
- *SSMF fiber*
  - L_s =100 km
  - γ = 1.27 $W^{-1}km^{-1}$
  - β_2=-21.7 ps^2/km
  - α_{dB}=0.22 dB/km
- F = 5dB
- ν = 193 THz

The optimum PSD does not depend on transmission distance

Comments

- One relevant feature of previous figure is that the optimum launch power (or signal PSD) is the same for every distance

Comments

- One relevant feature of previous figure is that the optimum launch power (or signal PSD) is the same for every distance

$$G_{Tx,opt} = \frac{3}{2^{\frac{4}{3}}} \left(\frac{\left(e^{2\alpha L_s} - 1\right) F h \nu \pi |\beta_2|}{\gamma^2 L_{eff} \log\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)} \right)^{\frac{1}{3}}$$

Comments

- One relevant feature of previous figure is that the optimum launch power (or signal PSD) is the same for every distance:

$$G_{Tx,opt} = \frac{3}{2^{\frac{4}{3}}} \left(\frac{\left(e^{2\alpha L_s} - 1\right) F h \nu \pi |\beta_2|}{\gamma^2 L_{eff} \log\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)} \right)^{\frac{1}{3}}$$

- For fixed amplifier noise figure and total bandwidth occupancy, the optimum launch power is independent of the number of spans (and consequently of the total link length).
- It indeed depends on fiber parameters (span length, fiber loss, dispersion, nonlinearity coefficient and effective length).

Realistic constellations

- To obtain capacity estimates for generic PM coherent formats in UT links, the standard formulas of capacity over AWGN [4], specific of each format, should be used, with the SNR derived from the generalized OSNR expression:

$$\mathrm{SNR} = \frac{B_n}{R_s}\mathrm{OSNR}_{\mathrm{NL}}$$

$$\mathrm{OSNR}_{\mathrm{NL}} = \frac{R_s}{B_n}\frac{G_{Tx,ch}}{N_s\left(e^{2\alpha L_s}-1\right)Fh\nu + \left(\frac{2}{3}\right)^3 \gamma^2 N_s L_{eff} G_{Tx,ch}^3 \frac{\ln\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)}{\pi|\beta_2|}}$$

[4] S. Benedetto and E. Biglieri, Principles of digital transmission: with wireless applications, New York: Kluwer,1999.

Hard and soft-decision capacity

HARD DECISION

$$C = 2\frac{1}{M}\sum_{a,b} P_{Y|X}(b|a)\log_2 \frac{P_{Y|X}(b|a)}{P_Y(b)}$$

SOFT DECISION

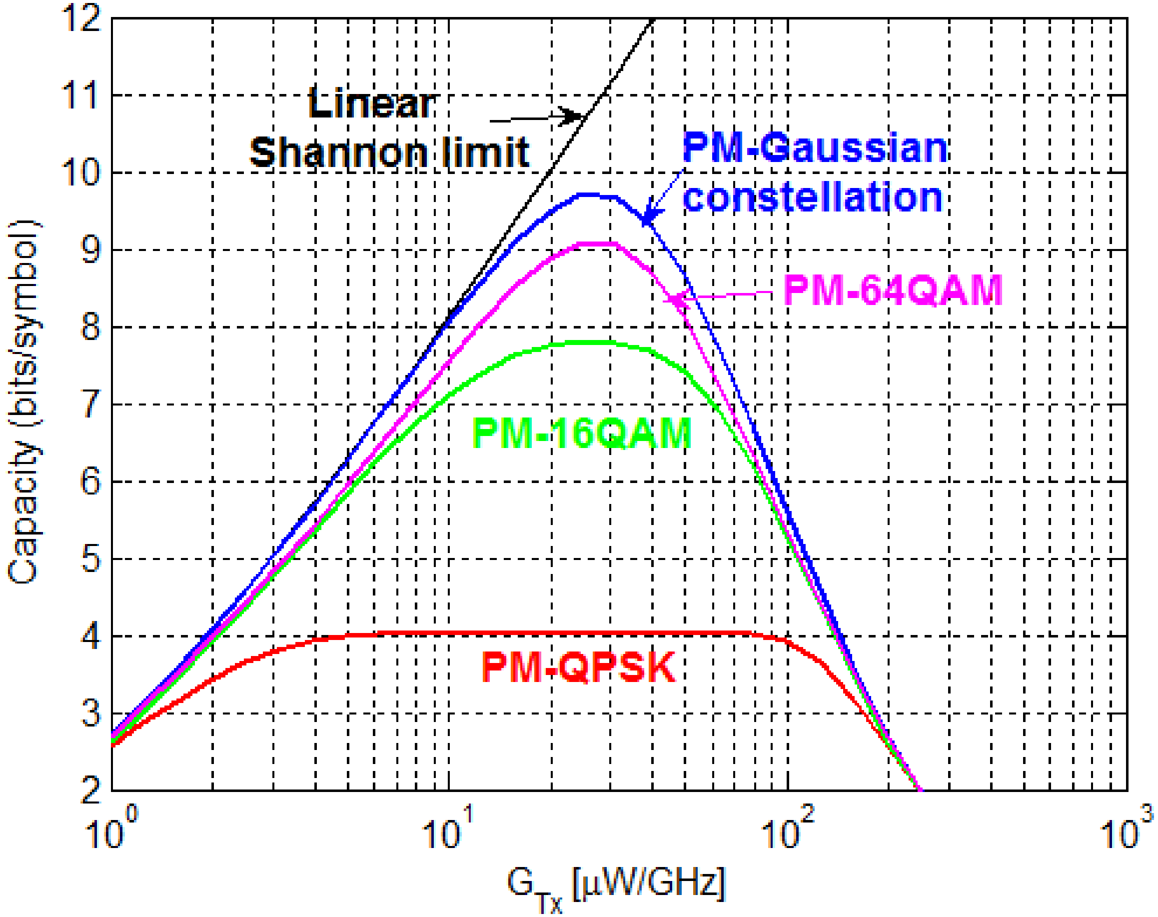
$$C = 2\frac{1}{M}\sum_{a\in X}\int p_{Y|X}(y|a)\log_2 \frac{p_{Y|X}(y|a)}{p_Y(y)}$$

- All symbols are assumed to have the same transmission probability
- $X=\{x_1,..,x_M\}$ is the input alphabet
- $Y=\{y_1,..,y_M\}$ is the hard-decision output alphabet
- y is the soft value at the output of the channel

- $P_{Y|X}(b|a)$=probability of receiving b when a has been transmitted
- $P_Y(b)$= probability of receiving each of the constellation symbols
- In an AWGN channel: $p_{Y|X}(y|a) = \frac{1}{\pi\sigma_N^2} e^{-\frac{d^2(a,y)}{\sigma_N^2}}$

EDFA amplification (1000 km)

Soft decision

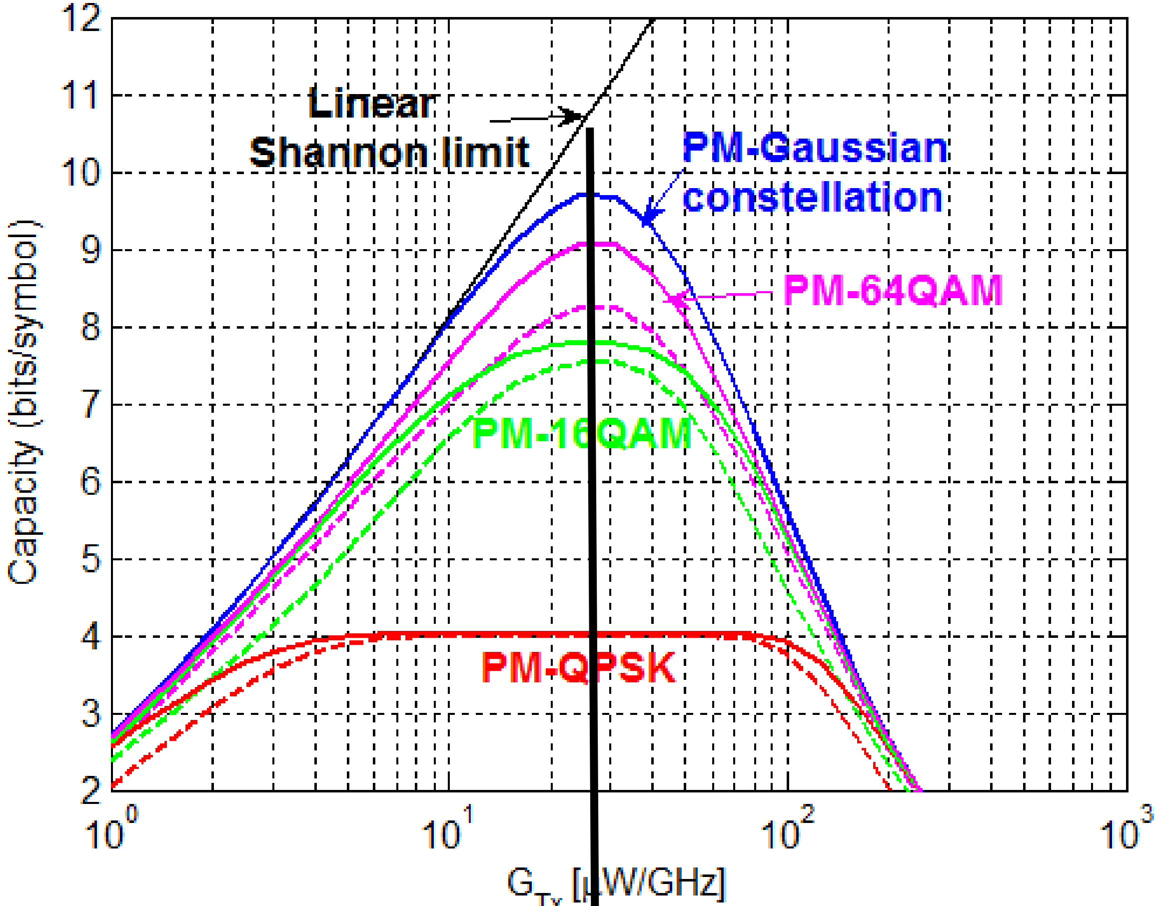


- SSMF fiber
- L_s=100 km
- N_s=10
- $\Delta f=R_s$
- B_{WDM}=4 THz
- ν=193 THz
- F=5 dB

	BER at maximum
PM-QPSK	$6.4\cdot10^{-8}$
PM-16QAM	$6.8\cdot10^{-3}$
PM-64QAM	$7.2\cdot10^{-2}$

EDFA amplification (1000 km)

Solid lines: soft decision
Dashed lines: hard decision



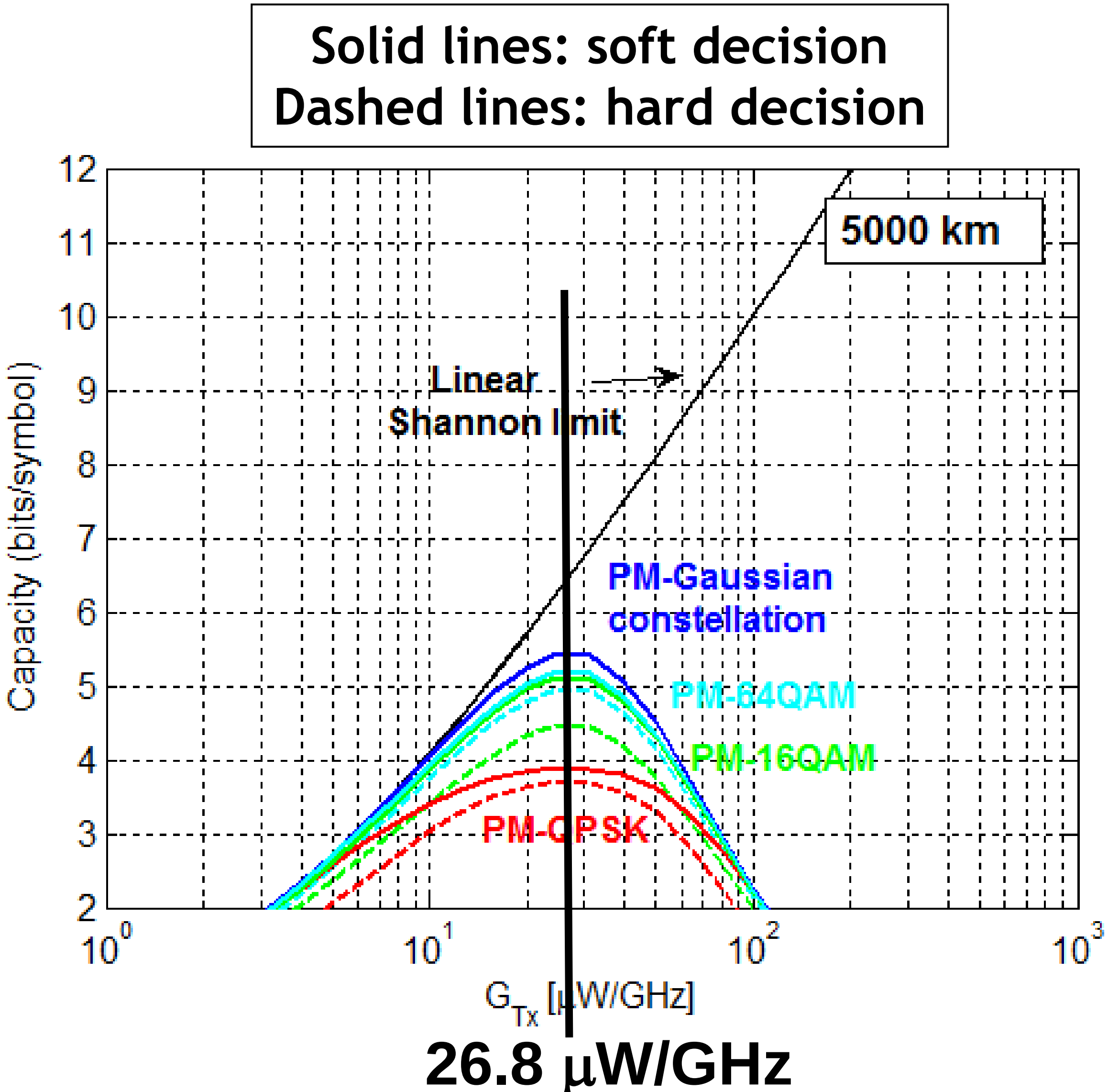
26.8 μW/GHz

- SSMF fiber
- L_s=100 km
- N_s=10
- $\Delta f=R_s$
- B_{WDM}=4 THz
- ν=193 THz
- F=5 dB

The optimum PSD does not depend on modulation format

	BER at maximum
PM-QPSK	$6.4\cdot10^{-8}$
PM-16QAM	$6.8\cdot10^{-3}$
PM-64QAM	$7.2\cdot10^{-2}$

EDFA amplification (5000 km)

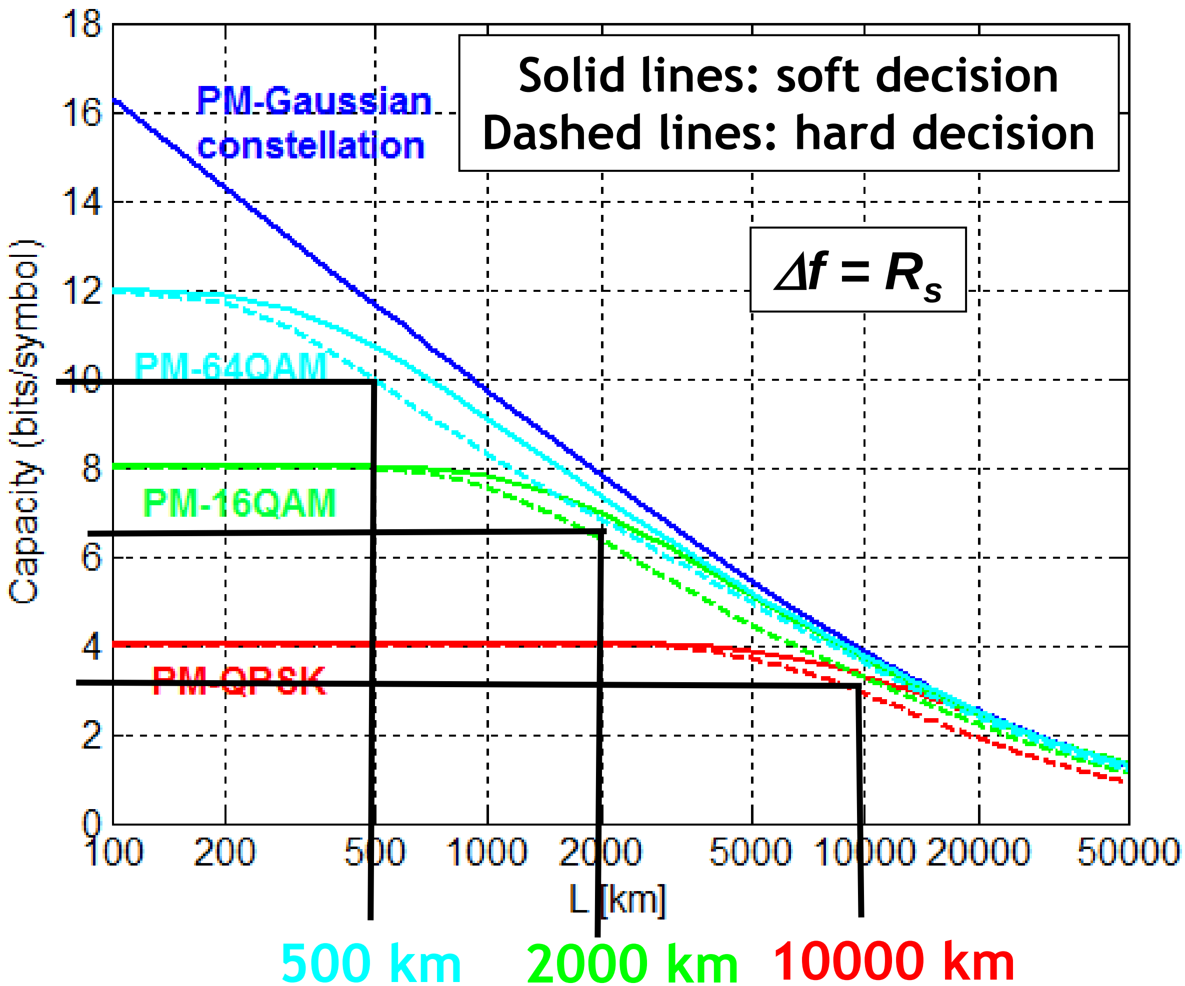


- SSMF fiber
- L_s=100 km
- N_s=50
- $\Delta f=R_s$
- B_{WDM}=4 THz
- ν=193 THz
- F=5 dB

The optimum PSD does not depend on transmission distance

SSMF with 100 km span length

- Maximum capacity (at optimum PSD) vs. distance

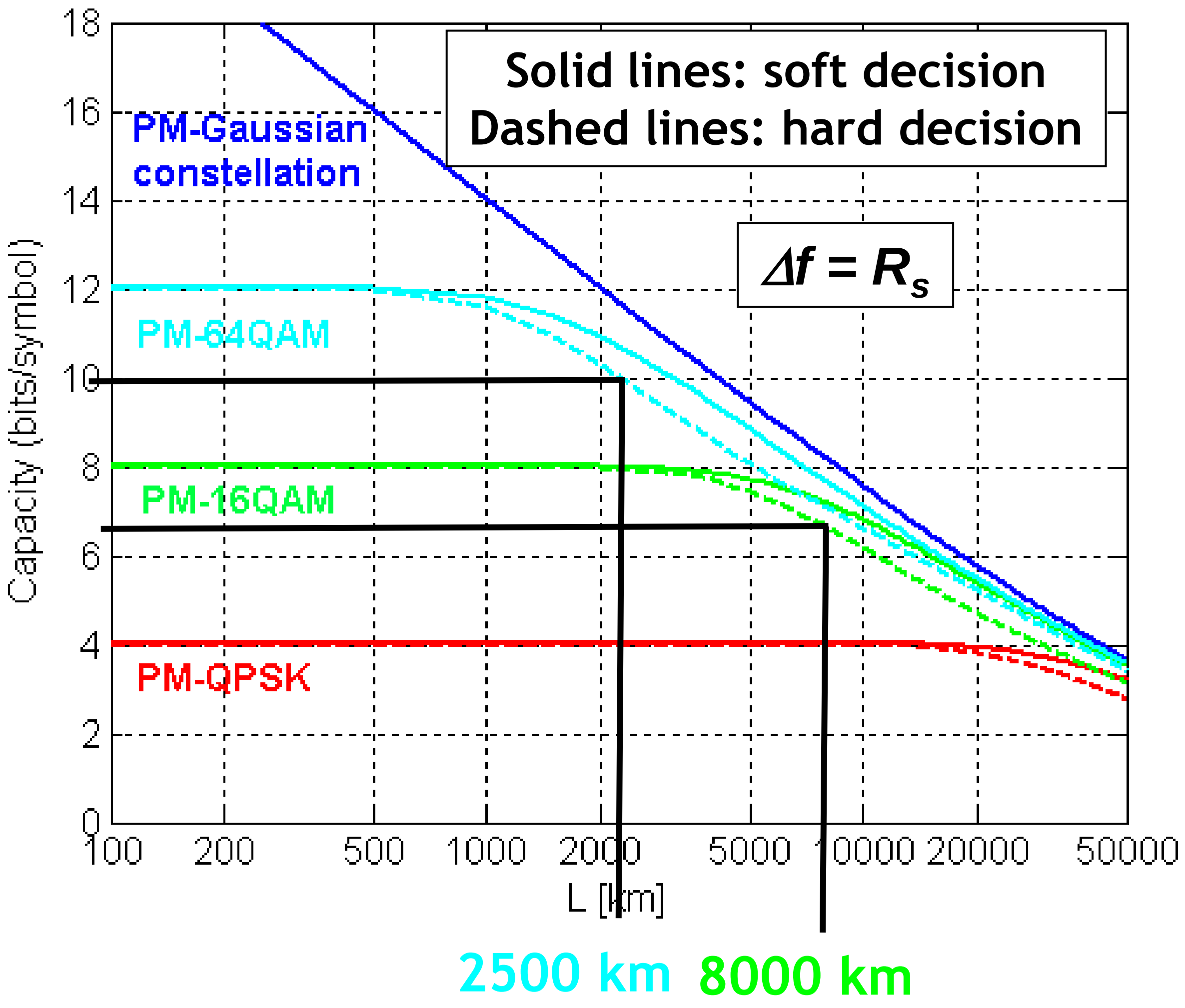


- Fixing the distance, the capacity penalty of each format with respect to its maximum corresponds to the required FEC overhead.
- 20% hard-FEC overhead

Trade-off between capacity and distance

PSCF with 50 km span length

- **Maximum capacity (at optimum PSD) vs. distance**



- *PSCF fiber*
  - γ = 1.0 $W^{-1}km^{-1}$
  - β_2=-26.2 ps^2/km
  - α_{dB}=0.18 dB/km

20% hard-FEC overhead

Conclusions

Non-linear propagation model for uncompensated transmission (validated both through simulations and experiments)

Analytical form of the channel capacity at the Nyquist limit.

Capacity is independent of the symbol-rate.
Optimum launch power spectral density is independent of link length.

Examples of application to uncompensated optical systems with EDFA amplification.

- The obtained results can be extended to the general case of non rectangular spectra and spacing larger than the symbol-rate.

Acknowledgments

This work was supported by the European Union within the EURO-FOS project, a Network of Excellence funded by the European Commission through the 7th ICT-Framework Programme.

This work was partially supported by CISCO Systems within a SRA contract.