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Advances in Modeling and Mitigation of Nonlinear Effects in Uncompensated Coherent Optical Transmission Systems

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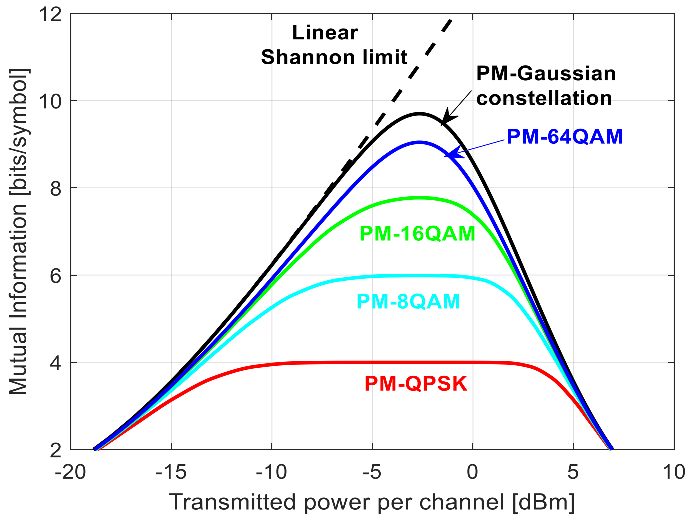
OPTCOM

Outline

- **Modeling of fiber nonlinearity**
  - Modeling approximations
  - The GN/EGN model family
  - Modeling of nonlinear propagation in different scenarios
    - Gaussian-constellations
    - Ultra-high symbol rates
    - Wideband optical systems
  - Closed-form formulas
- **Mitigation of fiber nonlinearity**
  - Non-linearity tolerant modulation formats
  - Symbol-rate optimization (SRO): model prediction vs. practical implementation
  - Digital back-propagation (DBP): model prediction vs. practical implementation
  - ML for nonlinearity mitigation

MODELING OF FIBER NONLINEARITY



- Goal: to predict the behaviour of a long-haul optical system/network in a reasonable amount of time.

Non-linear fiber propagation models

- Any form of analytical description of the non-linear behaviour of the optical fiber
- Example: non-linear Schrödinger equation (NLSE)

$$\boxed{\frac{\partial E(z,t)}{\partial z} = -\frac{\alpha}{2}E(z,t) - j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}E(z,t) + j\gamma\left|E(z,t)\right|^2 E(z,t)}$$

$$\boxed{\begin{aligned}\frac{\partial E_x(z,t)}{\partial z} &= -\frac{\alpha}{2}E_x(z,t) - j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}E_x(z,t) + j\gamma\left(\left|E_x(z,t)\right|^2 + B\left|E_y(z,t)\right|^2\right)E_x(z,t)\\ \frac{\partial E_y(z,t)}{\partial z} &= -\frac{\alpha}{2}E_y(z,t) - j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}E_y(z,t) + j\gamma\left(\left|E_y(z,t)\right|^2 + B\left|E_x(z,t)\right|^2\right)E_y(z,t)\end{aligned}}$$

G. P. Agrawal, Nonlinear Fiber Optics, 4th edition. Academic Press, 2007, Chapter 6.

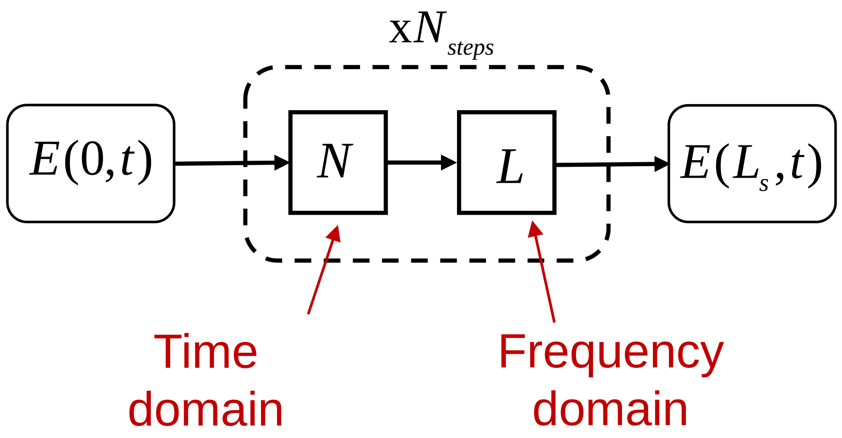
- Numerical integration within a Monte-Carlo simulation environment (e.g. using the split-step Fourier method – SSFM)

The Split-Step Fourier (SSFM) Method

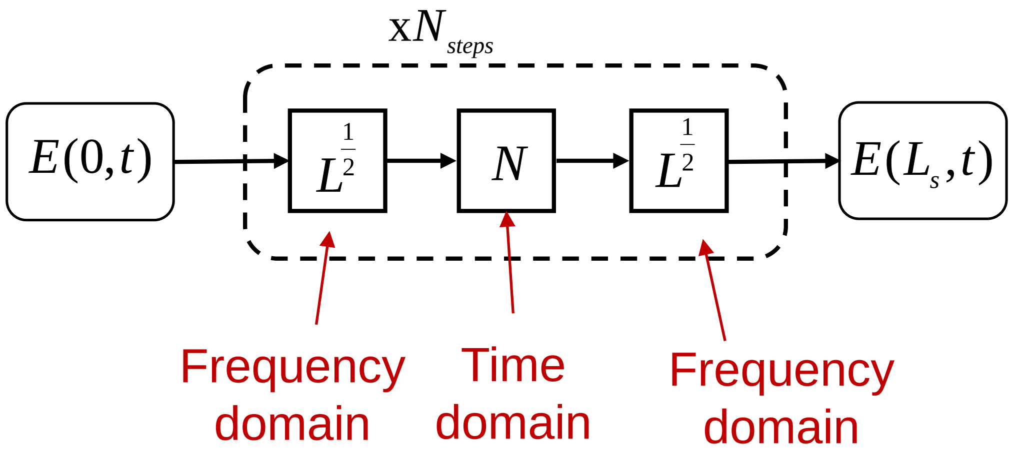
$$\frac{\partial E(z,t)}{\partial z} = \underbrace{\boxed{\left[-\frac{\alpha}{2} - j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\right]}}_{\boldsymbol{L}} E(z,t) + j\gamma \underbrace{\boxed{\left|E(z,t)\right|^2}}_{\boldsymbol{N}} E(z,t)$$

High complexity!

ASYMMETRIC SSFM



SYMMETRIC SSFM



- Goal: to find simpler yet accurate models in order to quantify the system impact of the fiber non-linear behaviour

Families of models

- Examples:
  - first order perturbation
  - higher-order perturbation
  - regular perturbation (RP, with variants)
  - logarithmic perturbation (LP, with variants)
  - time domain
  - frequency domain
  - Volterra-based
  - pulse-collision based
  - more classes and sub-classes based on specific assumptions and approximations…

- In this talk, I will focus on **frequency-domain RP first-order models**

Incoherent NLI acc.

Manakov equation

Perturbation approach

Modeling approximations

Signal Gaussianity

Locally white NLI

NLI as additive GN

Manakov equation

$$\begin{cases} \dfrac{\partial E_x(z,t)}{\partial z} = -j\dfrac{\beta_2}{2}\dfrac{\partial^2}{\partial t^2}E_x(z,t) - \dfrac{\alpha}{2}E_x(z,t) + j\gamma\dfrac{8}{9}\left[\left|E_x(z,t)\right|^2 + \left|E_y(z,t)\right|^2\right]E_x(z,t) \\ \dfrac{\partial E_y(z,t)}{\partial z} = -j\dfrac{\beta_2}{2}\dfrac{\partial^2}{\partial t^2}E_y(z,t) - \dfrac{\alpha}{2}E_y(z,t) + j\gamma\dfrac{8}{9}\left[\left|E_x(z,t)\right|^2 + \left|E_y(z,t)\right|^2\right]E_y(z,t) \end{cases}$$

- It’s based on an analytical average over the random evolution of the state-of-polarization (SOP) along the fiber
- It captures the non-linear effects of one polarization onto the other, but averages over the fast dynamic of SOP variations
- It neglects both linear and nonlinear effects of PMD

First-order regular perturbation

- <u>Assumptions:</u>
  - The signal propagates linearly from input to output
  - At each point along the fiber, it excites fiber nonlinearity and creates the NLI disturbance
  - At the end of the fiber, the linearly propagated signal and the NLI are summed (NLI noise can be represented as an additive noise term)

$$s_{WDM}^{NL}(t) = s_{WDM}(t) + \boxed{s_{NLI}(t)}$$

NON-LINEAR INTERFERENCE (NLI)

- In the framework of first-order perturbation analyses, the NLI power is proportional to P_{ch}^3:

$$\boxed{P_{NLI} = \eta P_{ch}^3}$$

  - where η is a coefficient that depends on the fiber parameters and the transmitted signal characteristics.

NLI additive Gaussian noise approximation

- Assumption:
  - the NLI at the output of the link can be represented as additive Gaussian noise, circular and independent of either the signal or ASE noise

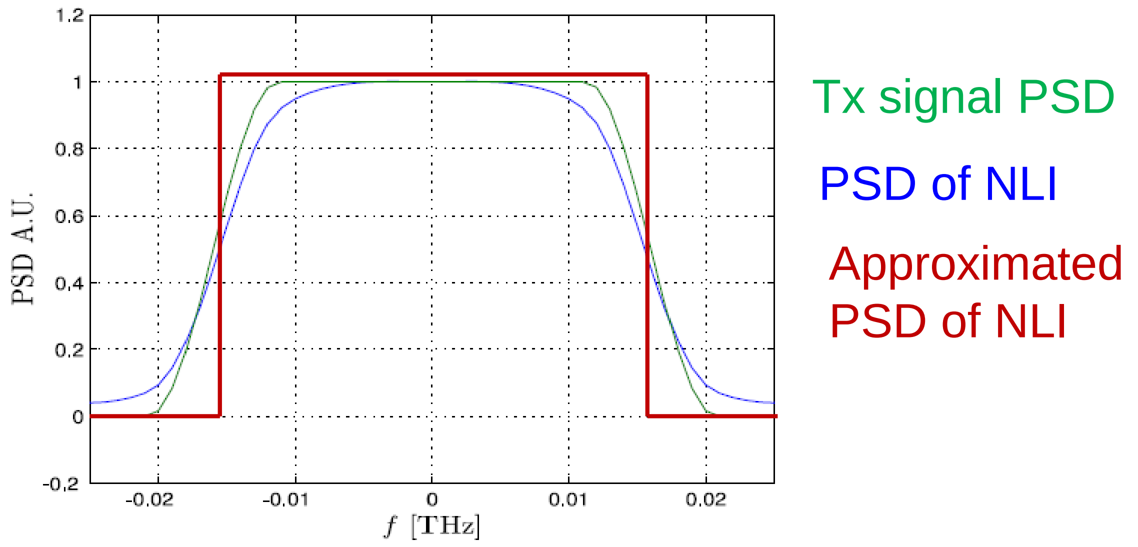
- Key implication: the channel performance can be characterized based on a modified "non-linear" OSNR:

$$\boxed{OSNR_{NL} = \frac{P_{ch}}{P_{ASE} + P_{NLI}}}$$

  - P_{ch} : power of channel under test
  - P_{ASE} : power of ASE noise
  - P_{NLI} is the power of NLI

Locally white NLI noise approximation

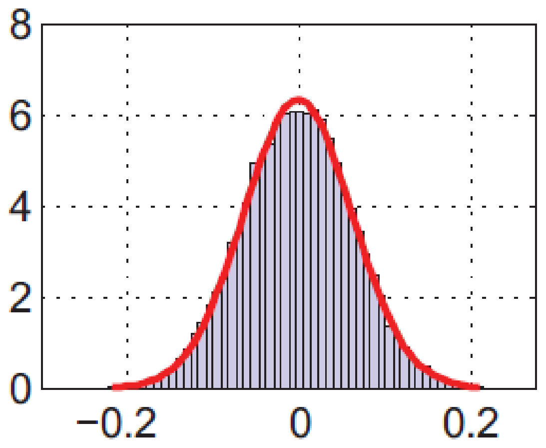
- <u>Assumption:</u>
  - the PSD of NLI is locally flat (over a single channel bandwidth)



- This assumption is acceptable for approximate system performance assessment.
- It should be removed for high-accuracy predictions.

The signal Gaussianity approximation

- <u>Assumption:</u>
  - the transmitted signal can be modeled as a stationary circular Gaussian noise, whose PSD is shaped as the PSD of the actually transmitted WDM channels.



500 km of SSMF fiber

- This approximation allows to drastically simplify the model derivation and strongly decreases the model final analytical complexity.

- Using this assumption, the impact of NLI is always overestimated for QAM transmission formats.

The incoherent NLI accumulation approximation

- Assumption:
  - the NLI produced in each span adds up incoherently (i.e., in power) at the receiver side:

$$G_{NLI}(f) \approx \sum_{n=1}^{N_{span}} G_{NLI}^{(n)}(f)$$

  - In reality, the NLI contributions should be added together coherently (i.e., at the field level) keeping both their amplitude and phase into account

- The accuracy of this approximation is quite poor at very low span count and at very low channel count.

The EGN-GN model family

EGN-model → GN-model → iGN-model

The EGN-GN model family

Assumption	EGN model	GN model	iGN model
Manakov equation	X	X	X
1st order regular perturbation	X	X	X
Signal Gaussianity		X	X
Incoherent NLI accumulation			X
NLI as additive Gaussian noise	*Approximations that can be applied to all models in order to simplify the computations*		
Locally white NLI			

- ***iGN*** – *P. Poggiolini et al., "Analytical Modeling of Nonlinear Propagation in Uncompensated Optical Transmission Links", IEEE Photon. Technol. Lett.* ***23****(11), p. 742 (2011).*
- ***GN*** – *P. Poggiolini "The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems,"J. Lightwave Technol.* ***30****(24), p.3857 (2012).*
- ***EGN*** – *A. Carena et al., "EGN model of non-linear fiber propagation," Opt. Exp.* ***22****(13), p. 16335, 2014.*

The simplest iGN closed-form solution

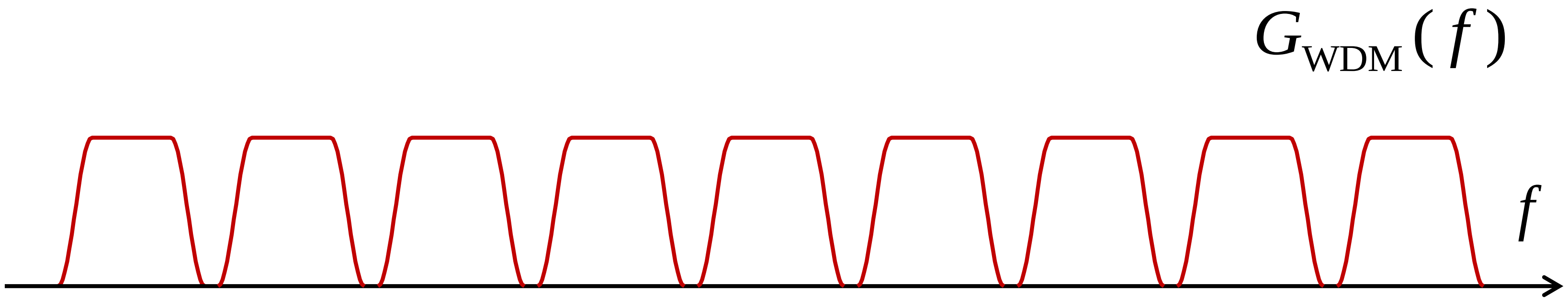
- All approximations listed in the previous slide, plus …
  - Equal spans
  - Equal channels (same power, same spectrum with bandwidth ~R_s)

$$\boxed{G_{\mathrm{NLI}}\left(f_c\right)=N_{span}\,\frac{16}{27}\,\frac{\gamma^2 L_{\mathrm{eff}}^2 P_{\mathrm{ch}}^3}{\pi\left|\beta_2\right|\alpha R_s^3}\,\operatorname{asinh}\left(\frac{\pi^2}{2\alpha}\left|\beta_2\right|R_s^2\left[N_{\mathrm{ch}}^2\right]^{\frac{R_s}{\Delta f}}\right)}$$

- The model equations become more and more complex, as well as more and more accurate, as the various assumptions are removed

The GN-model reference formula

$$G_{\mathrm{NLI}}(f) = \frac{16}{27} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{\mathrm{WDM}}(f_1) G_{\mathrm{WDM}}(f_2) G_{\mathrm{WDM}}(f_1 + f_2 - f) \cdot \left| \mu(f_1, f_2, f) \right|^2 \, \mathrm{d}f_1 \mathrm{d}f_2$$



FWM efficiency of the whole link

- For identical spans with lumped amplification:

$$\left| \mu(f_1, f_2, f) \right|^2 = \gamma^2 L_{\mathrm{eff}}^2 \left| \frac{1 - e^{-2\alpha L_s} e^{j4\pi^2 \beta_2 L_s (f_1 - f)(f_2 - f)}}{1 - j2\pi^2 \beta_2 \alpha^{-1} (f_1 - f)(f_2 - f)} \right|^2 \frac{\sin^2\left(2 N_s \pi^2 (f_1 - f)(f_2 - f) \beta_2 L_s\right)}{\sin^2\left(2\pi^2 (f_1 - f)(f_2 - f) \beta_2 L_s\right)}$$

The enhanced GN (EGN) model

- The EGN model consists of the GN model and of a “correction” term:

$$G_{\mathrm{NLI}}^{\mathrm{EGN}}(f) = G_{\mathrm{NLI}}^{\mathrm{GN}}(f) - G_{\mathrm{NLI}}^{\mathrm{corr}}(f)$$

$$G_{\mathrm{NLI}}^{\mathrm{corr}}(f) = G_{\mathrm{SPM}}^{\mathrm{corr}}(f) + G_{\mathrm{X1\text{-}XPM}}^{\mathrm{corr}}(f) + \sum_{i=2}^{4} G_{\mathrm{X}i}^{\mathrm{corr}}(f) + \sum_{i=1}^{3} G_{\mathrm{M}i}^{\mathrm{corr}}(f)$$

SPM XPM FWM

$$G_{\mathrm{SPM}}^{\mathrm{corr}}(f) = P_m^3\left[\Phi_m\,\kappa_m^{\mathrm{SPM}}(f) + \Psi_m\,\varsigma_m^{\mathrm{SPM}}(f)\right]$$

$$\varsigma_m^{\mathrm{SPM}}(f) = \frac{16}{81} R_m \int_{f_m-B_m/2}^{f_m+B_m/2} df_1 \int_{f_m-B_m/2}^{f_m+B_m/2} df_2 \int_{f_m-B_m/2}^{f_m+B_m/2} df_1' \int_{f_m-B_m/2}^{f_m+B_m/2} df_2' \cdot$$
$$s_m(f_1)s_m(f_2)s_m^*(f_1+f_2-f)s_m^*(f_1')s_m^*(f_2')s_m(f_1'+f_2'-f)\cdot$$
$$\mu(f_1,f_2,f)\,\mu^*(f_1',f_2',f)$$

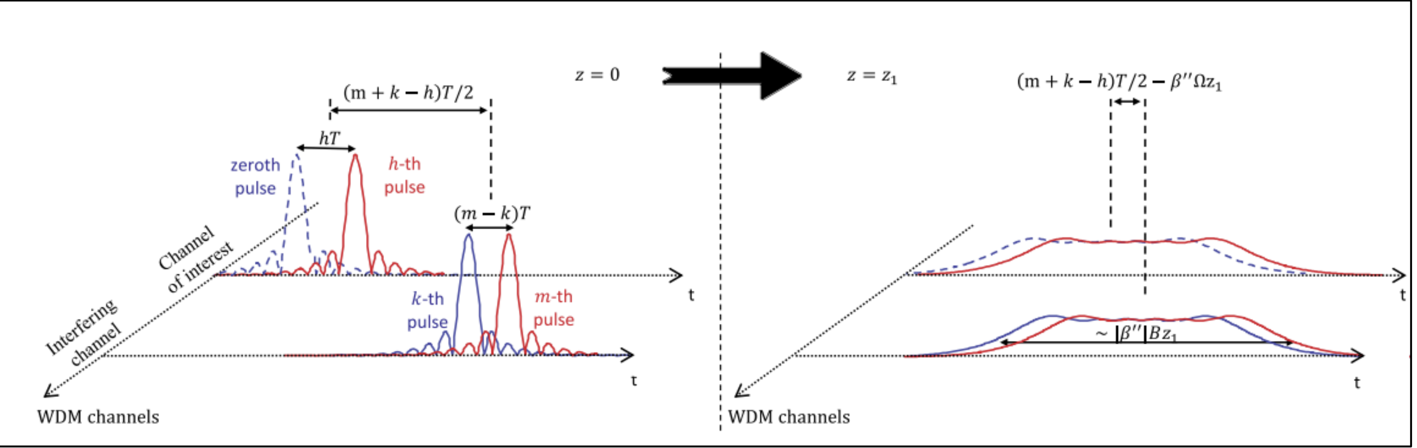
$$\kappa_m^{\mathrm{SPM}}(f) = \frac{80}{81} R_m^2 \int_{f_m-B_m/2}^{f_m+B_m/2} df_1 \int_{f_m-B_m/2}^{f_m+B_m/2} df_2 \int_{f_m-B_m/2}^{f_m+B_m/2} df_2' \cdot$$
$$|s_m(f_1)|^2 s_m(f_2)s_m^*(f_2')s_m^*(f_1+f_2-f)s_m(f_1+f_2'-f)\cdot$$
$$\mu(f_1,f_2,f)\,\mu^*(f_1,f_2',f)$$
$$+\frac{16}{81} R_m^2 \int_{f_m-B_m/2}^{f_m+B_m/2} df_1 \int_{f_m-B_m/2}^{f_m+B_m/2} df_2 \int_{f_m-B_m/2}^{f_m+B_m/2} df_2' \cdot$$
$$|s_m(f_1+f_2-f)|^2 s_m(f_1)s_m(f_2)s_m^*(f_1+f_2-f_2')s_m^*(f_2')\cdot$$
$$\mu(f_1,f_2,f)\,\mu^*(f_1+f_2-f_2',f_2',f)$$

Time-domain models

- Very similar to the EGN model in terms of accuracy and complexity
- Able to predict PPRN and temporal correlations.
- Nonlinear interference is described as an inter-symbol interference (ISI), predicting the contribution of the various ISI terms.

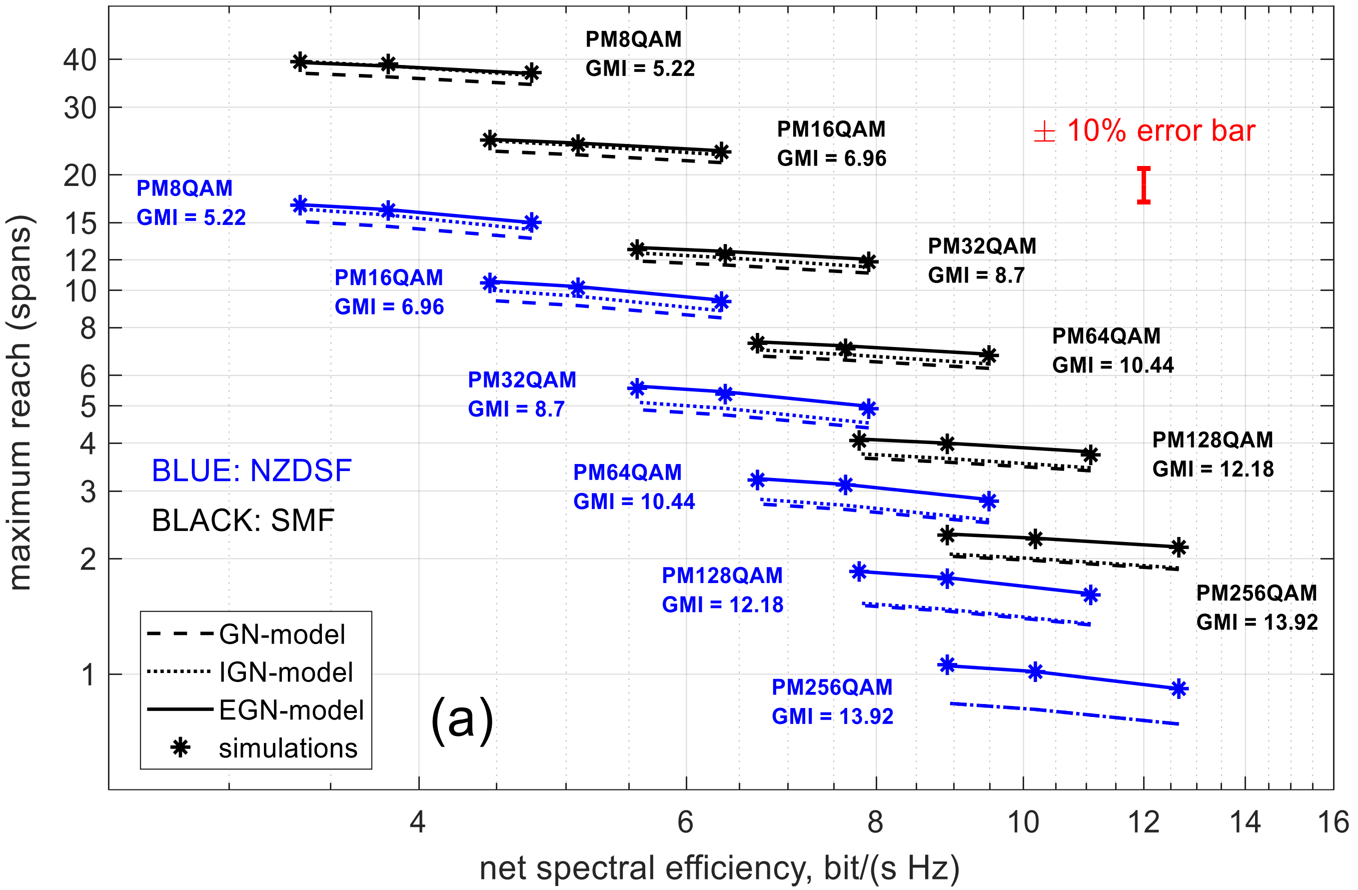
R. Dar, M. Feder, A. Mecozzi, M. Shtaif, " Properties of nonlinear noise in long, dispersion-uncompensated fiber links," Opt. Express 21(22), p. 25685 (2013)

P. Serena, A. Bononi, "A time-domain extended Gaussian noise model," J. Lightwave Technol. 33(7), p. 1459 (2015)



R. Dar, M. Feder, A. Mecozzi, M. Shtaif, "Pulse collision picture of inter-channel nonlinear interference noise in fiber-optic communications," J. Lightw. Technol. 34, p. 593 (2016)

Comparison between different approximations



- System data:
  - Symbol rate R_s=64GBaud
  - 15 WDM channels
  - span length 100km
  - EDFA noise figure 6dB

GN and iGN models always underestimate the performance, with a better accuracy for a high number of spans, where the accumulated dispersion is higher.

A. Bononi, R. Dar , M. Secondini, P. Serena , P. Poggiolini, "Fiber Nonlinearity and Optical System Perfomance", in Springer Handbook of Optical Networks, Springer International Publishing, 2020.

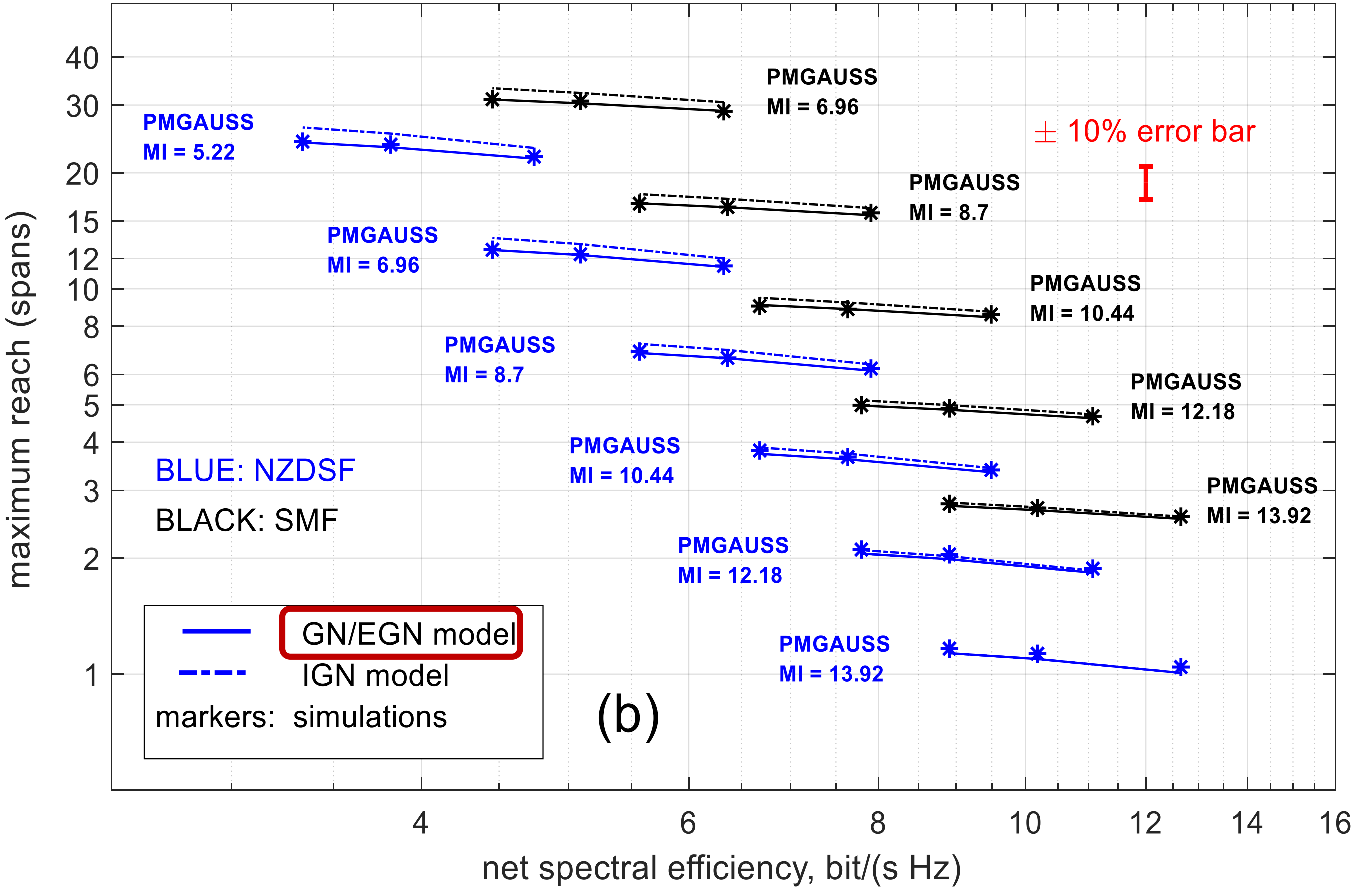
Modeling of nonlinear propagation in different scenarios

High symbol rate

Ultra wide-band systems

Gaussian-like constellations

Gaussian-like constellations



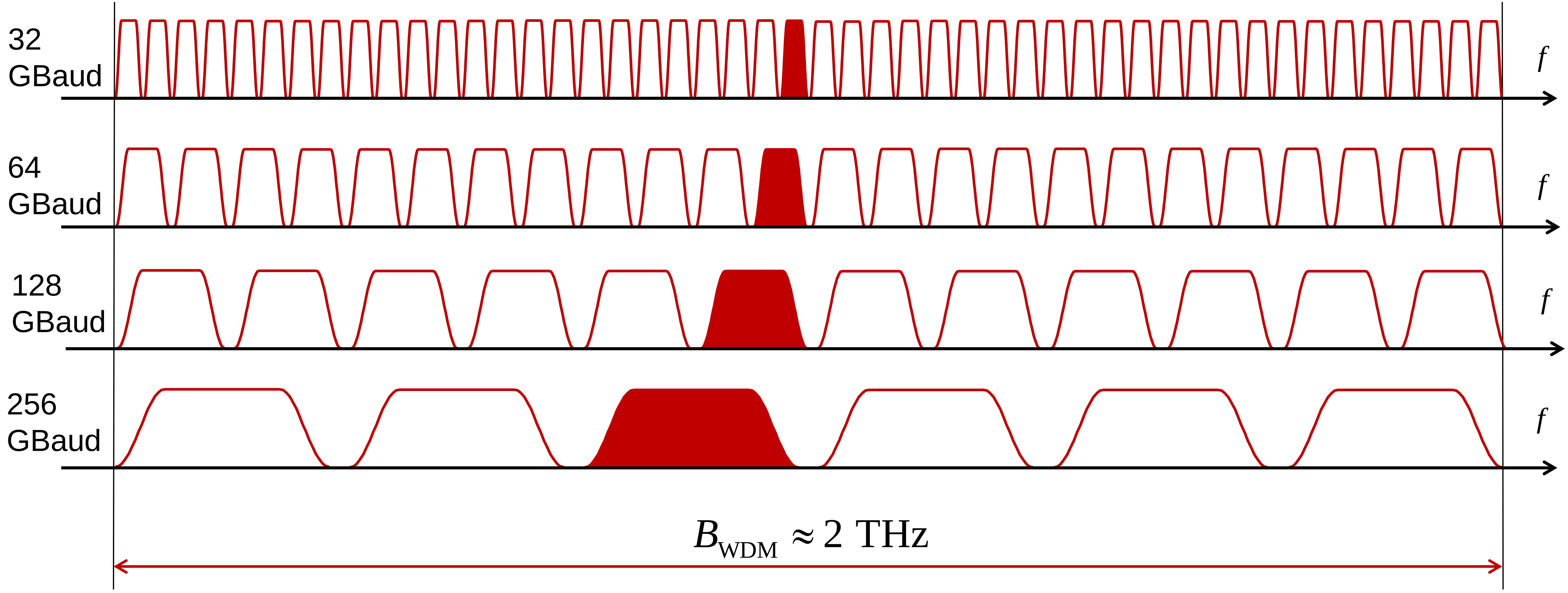
- System data:
  - Symbol rate R_s=64GBaud
  - 15 WDM channels
  - span length 100km
  - EDFA noise figure 6dB

GN and EGN model coincide

The accuracy of the incoherent GN model is higher than for QAM constellations

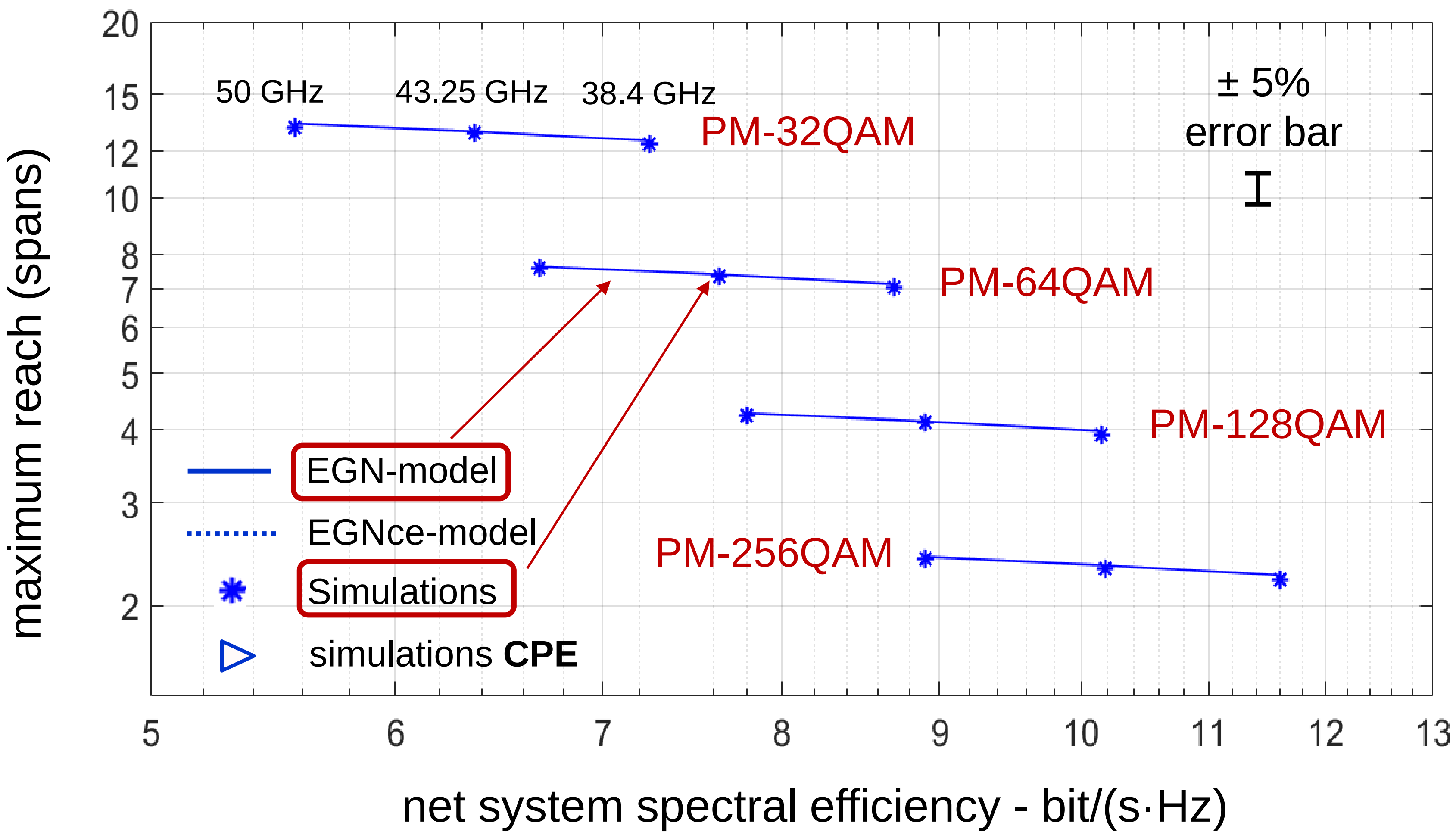
A. Bononi, R. Dar , M. Secondini, P. Serena , P. Poggiolini, ”Fiber Nonlinearity and Optical System Perfomance”, in Springer Handbook of Optical Networks, Springer International Publishing, 2020.

Increasing the symbol rate …

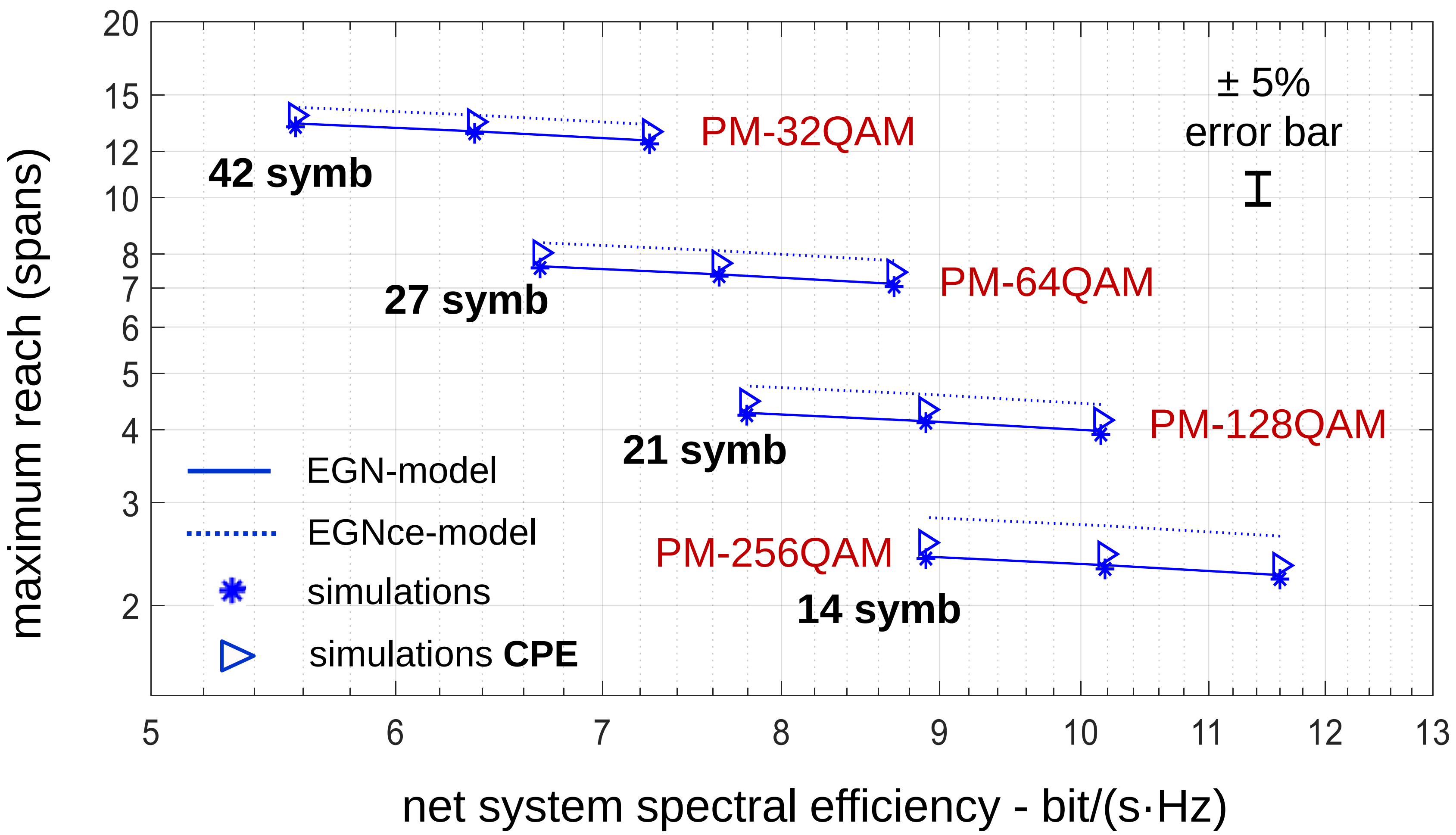


P. Poggiolini et al., "Non-Linearity Modeling at Ultra-High Symbol Rates," Proc. Of OFC 2018, San Diego (USA), Mar. 2018.

32 Gbaud - 48 channels - SMF - 100km spans

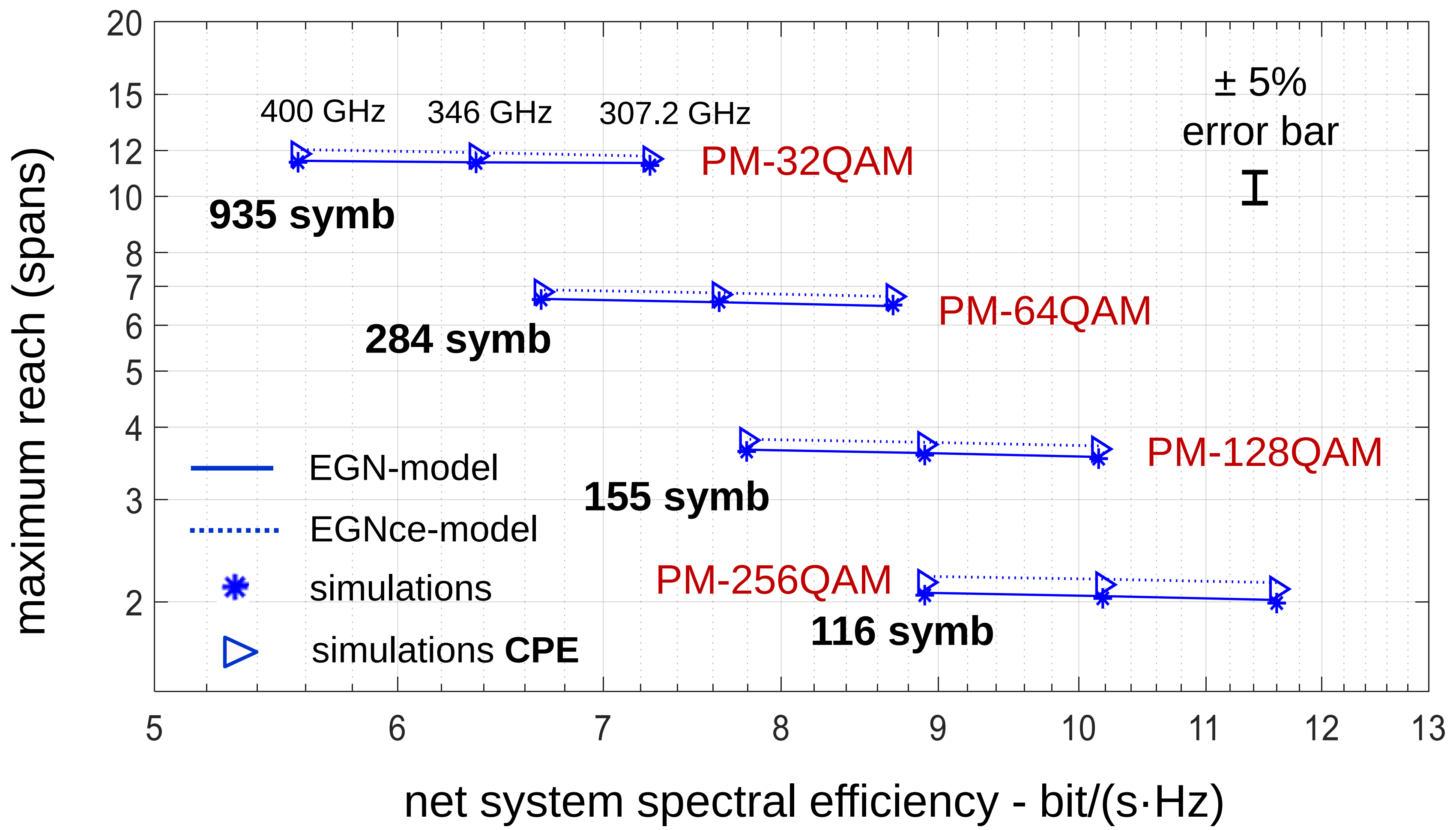


32 Gbaud - 48 channels - SMF - 100km spans



- EGNce-model:
  - Modified EGN model, where the correlated NLPN phase noise is ideally taken out
  - It corresponds to the EGN model calculated as if a constant envelope constellation, was transmitted.

256 Gbaud - 6 channels - SMF - 100km spans



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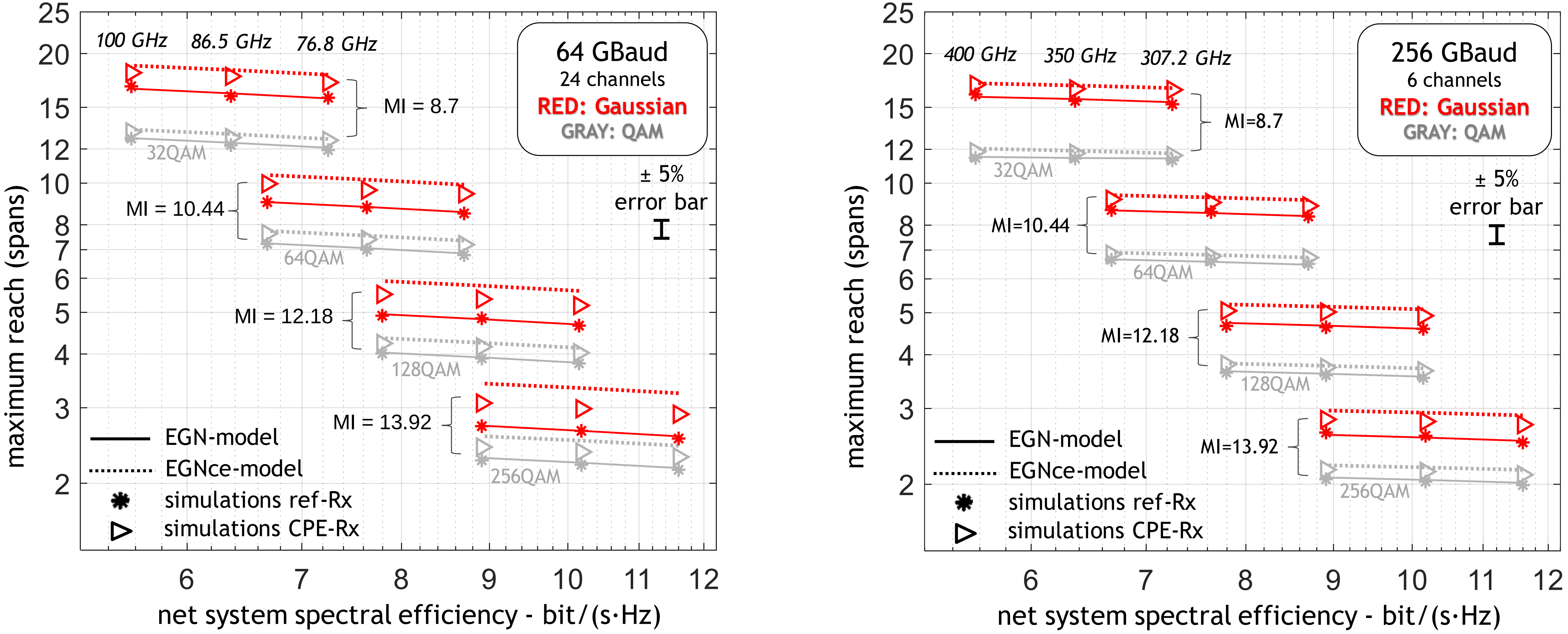
Gaussian constellations – 64 and 256 Gbaud



P. Poggiolini et al., "Non-Linearity Modeling for Gaussian-Constellation Systems at Ultra-High Symbol Rates," Proc. Of ECOC 2018, Rome (Italy), Sep. 2018.

Key take-aways, so far …

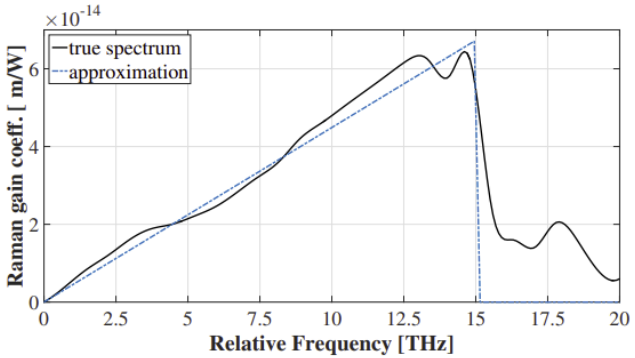
- The EGN model appears to be extremely reliable, across all the explored parameter space (ultra-high symbol rates, QAM and Gaussian constellations).

- It coincides with the much computationally simpler GN model for Gaussian constellations.

- Going towards higher symbol rates, the NLPN decreases, while its higher for Gaussian-like constellations, as shown by the EGNce-model results.

Non-Linearity Modeling for Wide-Band Optical Systems

- The effect of inter-channel stimulated Raman scattering (ISRS) has to be included in the nonlinear models when analyzing ultra-wideband transmission.



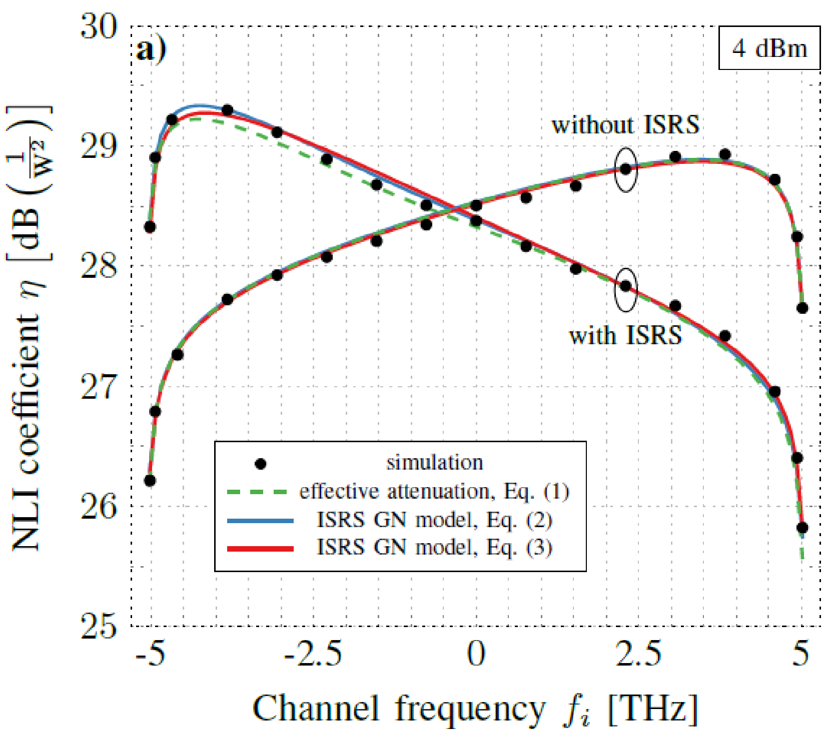
- Assuming that the temporal gain dynamics of ISRS are negligible, ISRS can be modeled as a frequency- and distance-dependent signal power profile ρ (z; f), which is obtained by solving the continuous-wave Raman equations (D. N. Christodoulides et al., PTL, 8, (12), p.1722,1996).

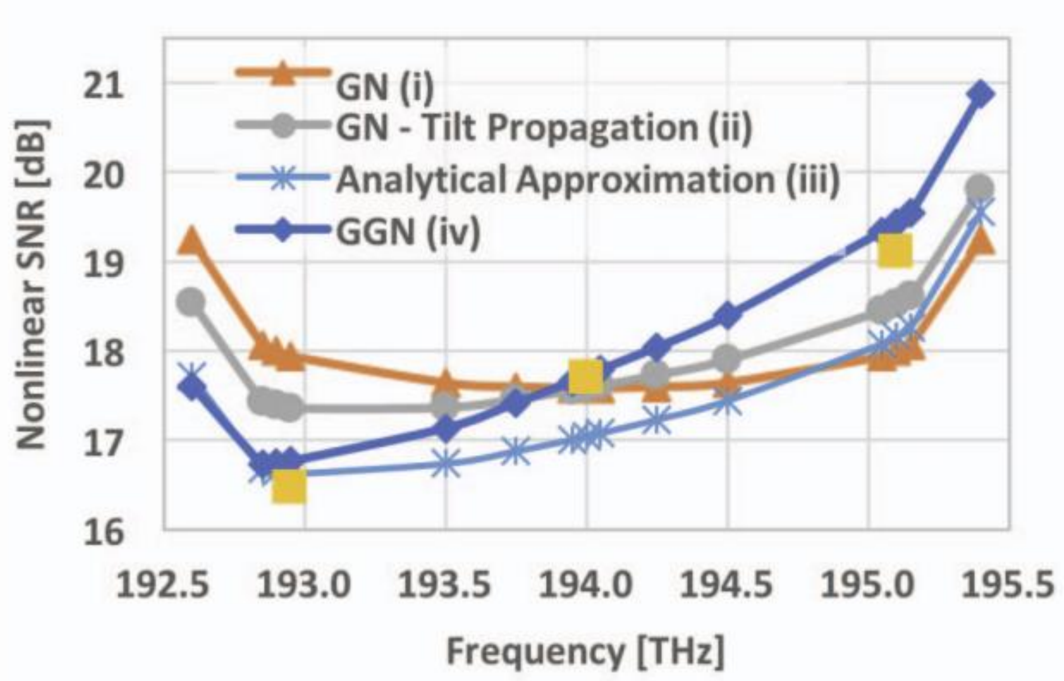
- Approaches to include ISRS in the conventional GN model can be divided into two groups:
  1. Effective attenuation approach: ρ (z; f) is approximated with exponential decays, that have modified attenuation coefficients or effective lengths.

     D. Semrau et al., OpEx, 25, (12), p.13024, 2017 *M. Cantono et al., OFC 2018, San Diego, M1D.2*
  2. ISRS GN model / Generalized GN model: the conventional GN model is re-derived, based on the exact signal power profile.

     I. Roberts et al, JLT 35, (23), p. 5237, 2017 *M. Cantono et al., JLT 36, (15), p. 3131, 2018*
     D. Semrau et al., JLT 36, (14), p.3046, 2018 *D. Semrau et al., ECOC 2018, Tu4G.6*

Model validations





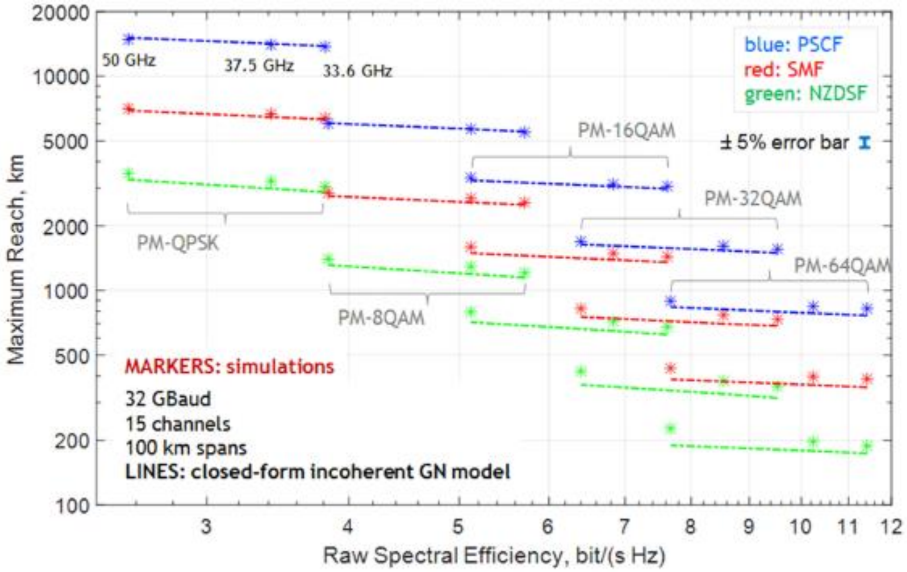
D. Semrau, R. I. Killey, and P. Bayvel, "Overview and comparison of nonlinear interference modelling approaches in ultra-wideband optical transmission systems," ICTON 2019).

M. Cantono et al.: Modelling the impact of SRS on NLI generation in commercial equipment: an experimental investigation, Proc. OFC 2018, San Diego, United States, March, M1D.2

The simplest iGN-model formula

$$G_{\mathrm{NLI}}\left(f_c\right) = N_{span}\,\frac{16}{27}\,\frac{\gamma^2 L_{\mathrm{eff}}^2 P_{\mathrm{ch}}^3}{\pi\left|\beta_2\right|\alpha R_s^3}\,\mathrm{asinh}\left(\frac{\pi^2}{2\alpha}\left|\beta_2\right| R_s^2\left[N_{\mathrm{ch}}^2\right]^{\frac{R_s}{\Delta f}}\right)$$

Closed-form models (CFM)



P. Poggiolini, Y. Jiang, JLT 35(3), p.458, 2017.

- Assumptions:
  - Signal Gaussianity
  - Incoherent NLI accumulation
  - NLI as additive Gaussian noise
  - Locally white NLI
  - Equal spans
  - Equal channels

EGN model correction term

$$G_{\mathrm{NLI}}^{\mathrm{EGN}}(f) \approx G_{\mathrm{NLI}}^{\mathrm{GN}}(f) + \underrightarrow{G}{}_{\mathrm{NLI}}^{\mathrm{corr}}$$

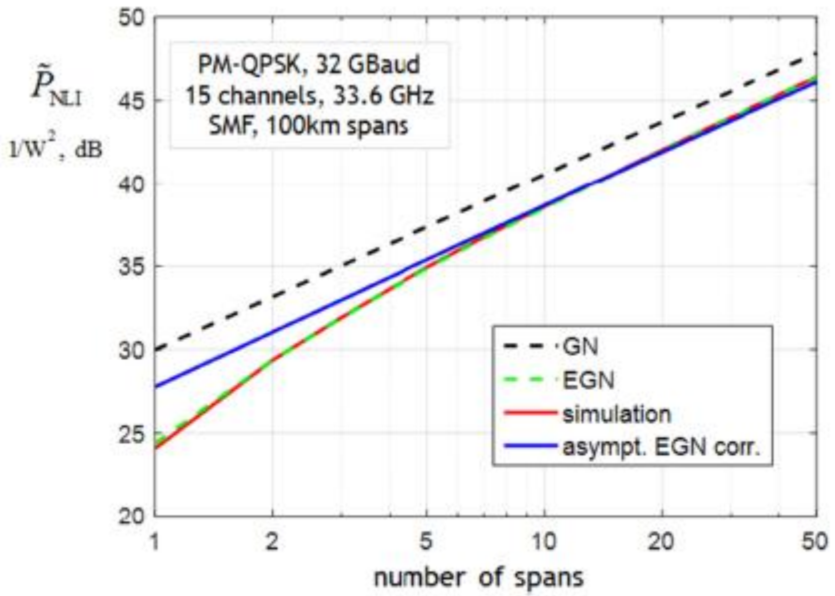
Different channels

$$\underrightarrow{G}{}_{\mathrm{NLI}}^{\mathrm{corr}} = \frac{40}{81} \frac{\gamma^2 P_m N_s \bar{L}_{\mathrm{eff}}^2}{R_m \pi \beta_2 \bar{L}_s} \cdot \left(\sum_{\substack{n=1 \\ n \neq m}}^{N_{\mathrm{ch}}} \Phi_n \frac{P_n^2}{R_n |f_n - f_m|} + \Phi_m \frac{2P_m^2}{R_m^2} \right)$$

Equal channels

$$\underrightarrow{G}{}_{\mathrm{NLI}}^{\mathrm{corr}} = \frac{80}{81} \Phi \frac{\gamma^2 \bar{L}_{\mathrm{eff}}^2 P_{\mathrm{ch}}^3 N_s}{R^2 \Delta f \pi \beta_2 \bar{L}_s} \left[\mathrm{HN}\left([N_{\mathrm{ch}} - 1]/2 \right) + \frac{\Delta f}{R} \right]$$

format	Φ
PM-BPSK	1
PM-QPSK	1
PM-8QAM	2/3
PM-16QAM	17/25
PM-32QAM	69/100
PM-64QAM	13/21
PM-∞-QAM	3/5
PM-Gaussian	0



P. Poggiolini and Y. Jiang, "Recent Advances in the Modeling of the Impact of Nonlinear Fiber Propagation Effects on Uncompensated Coherent Transmission Systems," J. Lightw. Technol. 35(3), p.458, 2017.

Including ISRS

$$\eta_n(f_i) \approx \frac{4}{9}\frac{\gamma^2}{B_i^2}\frac{\pi n^{1+\epsilon}}{\phi_i\bar{\alpha}(2\alpha+\bar{\alpha})}$$

$$\cdot\left[\frac{T_i-\alpha^2}{a}\text{asinh}\left(\frac{\phi_i B_i^2}{\pi a}\right)+\frac{A^2-T_i}{A}\text{asinh}\left(\frac{\phi_i B_i^2}{\pi A}\right)\right]$$

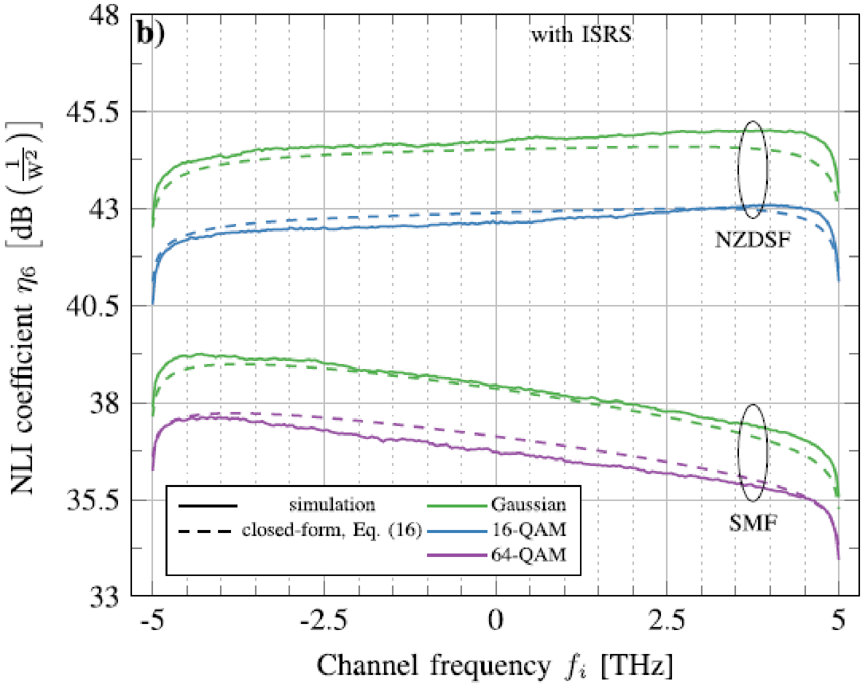
$$+\frac{32}{27}\sum_{k=1,k\neq i}^{N_{\text{ch}}}\left(\frac{P_k}{P_i}\right)^2\frac{\gamma^2}{B_k}$$

$$\left\{\frac{n+\frac{5}{6}\Phi}{\phi_{i,k}\bar{\alpha}(2\alpha+\bar{\alpha})}\cdot\left[\frac{T_k-\alpha^2}{\alpha}\text{atan}\left(\frac{\phi_{i,k}B_i}{\alpha}\right)\right.\right.$$

$$\left.+\frac{A^2-T_k}{A}\text{atan}\left(\frac{\phi_{i,k}B_i}{A}\right)\right]+\frac{5}{3}\frac{\Phi\pi\tilde{n}T_k}{|\phi|B_k^2\alpha^2A^2}$$

$$\left.\left[(2|\Delta f|-B_k)\log\left(\frac{2|\Delta f|-B_k}{2|\Delta f|+B_k}\right)+2B_k\right]\right\},$$

with $\phi_i = \frac{3}{2}\pi^2(\beta_2+2\pi\beta_3 f_i)$, $T_k = (\alpha+\bar{\alpha}-P_{\text{tot}}C_r f_k)^2$,
$\Delta f = f_k - f_i$, $\phi_{i,k} = -4\pi^2(f_k-f_i)[\beta_2+\pi\beta_3(f_i+f_k)]$,
$A = \alpha+\bar{\alpha}$ and coherence factor ϵ.



D. Semrau, R. I. Killey, P. Bayvel, "A Closed-Form Approximation of the Gaussian Noise Model in the Presence of Inter-Channel Stimulated Raman Scattering," J. Lightw. Technol. 37(9), p. 1924 (2019).

OPTCOM

Including ISRS

$$G_{\mathrm{NLI}}^{\mathrm{Rx}}\left(f_{\mathrm{CUT}}\right)=\sum_{n=1}^{N_{\mathrm{span}}}\left(G_{\mathrm{NLI}}^{(n)}\left(f_{\mathrm{CUT}}\right)\prod_{k=n+1}^{N_{\mathrm{span}}}\Gamma^{(k)}\left(f_{\mathrm{CUT}}\right)\cdot e^{-2\cdot\alpha^{(k)}\left(f_{\mathrm{CUT}}\right)\cdot L_{\mathrm{span}}^{(k)}}\right) \tag{1}$$

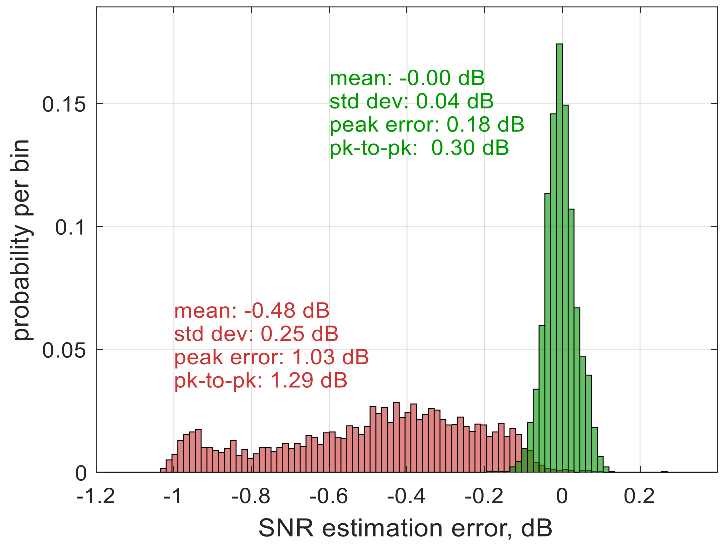
$$G_{\mathrm{NLI}}^{(n)}(f_{\mathrm{CUT}})=\frac{16}{27}(\gamma^{(n)})^2\,\Gamma^{(n)}(f_{\mathrm{CUT}})\cdot e^{-2\alpha^{(n)}(f_{\mathrm{CUT}})\cdot L_{\mathrm{span}}^{(n)}}\cdot\bar{G}_{\mathrm{CUT}}^{(n)}\cdot\left(\rho_{\mathrm{CUT}}^{(n)}\cdot[\bar{G}_{\mathrm{CUT}}^{(n)}]^2 I_{\mathrm{CUT}}^{(n)}+\sum_{n_{\mathrm{ch}}=1\ ,\ n_{\mathrm{ch}}\neq n_{\mathrm{CUT}}^{(n)}}^{N_{\mathrm{ch}}^{(n)}}2\rho_{n_{\mathrm{ch}}}^{(n)}\cdot[\bar{G}_{n_{\mathrm{ch}}}^{(n)}]^2 I_{n_{\mathrm{ch}}}^{(n)}\right) \tag{2}$$

$$I_{\mathrm{CUT}}^{(n)}=\frac{1}{2\pi\left|\bar{\beta}_{2,\mathrm{CUT}}^{(n)}\right|\cdot 2\alpha^{(n)}\left(f_{\mathrm{CUT}}\right)}\cdot\operatorname{asinh}\left(\frac{\pi^2}{2}\left|\frac{\bar{\beta}_{2,\mathrm{CUT}}^{(n)}}{2\alpha^{(n)}\left(f_{\mathrm{CUT}}\right)}\right|R_{\mathrm{CUT}}^2\right) \tag{3}$$

$$I_{n_{\mathrm{ch}}}^{(n)}=\frac{\operatorname{asinh}\left(\pi^2\left|\frac{\bar{\beta}_{2,n_{\mathrm{ch}}}^{(n)}}{2\alpha^{(n)}\left(f_{n_{\mathrm{ch}}}^{(n)}\right)}\right|\left[f_{n_{\mathrm{ch}}}^{(n)}-f_{\mathrm{CUT}}+\frac{R_{n_{\mathrm{ch}}}^{(n)}}{2}\right]R_{\mathrm{CUT}}\right)-\operatorname{asinh}\left(\pi^2\left|\frac{\bar{\beta}_{2,n_{\mathrm{ch}}}^{(n)}}{2\alpha^{(n)}\left(f_{n_{\mathrm{ch}}}^{(n)}\right)}\right|\left[f_{n_{\mathrm{ch}}}^{(n)}-f_{\mathrm{CUT}}-\frac{R_{n_{\mathrm{ch}}}^{(n)}}{2}\right]R_{\mathrm{CUT}}\right)}{4\pi\left|\bar{\beta}_{2,n_{\mathrm{ch}}}^{(n)}\right|\cdot 2\alpha^{(n)}\left(f_{n_{\mathrm{ch}}}^{(n)}\right)} \tag{4}$$

$$\bar{\beta}_{2,\mathrm{CUT}}^{(n)}=\beta_2^{(n)}+\pi\beta_3^{(n)}\left[2f_{\mathrm{CUT}}-2f_c^{(n)}\right]\quad,\quad\bar{\beta}_{2,n_{\mathrm{ch}}}^{(n)}=\beta_2^{(n)}+\pi\beta_3^{(n)}\left[f_{n_{\mathrm{ch}}}^{(n)}+f_{\mathrm{CUT}}-2f_c^{(n)}\right] \tag{5}$$

$$\boxed{\begin{aligned}\rho_{n_{\mathrm{ch}}}^{(n)}=&\left(1+a_{19}\cdot r_{\mathrm{CUT}}^{a_{20}}+a_{21}\cdot\left(r_{n_{\mathrm{ch}}}^{(n)}\right)^{a_{22}}\right)\cdot\left\{a_1+a_2\cdot\left(\Phi_{n_{\mathrm{ch}}}^{(n)}\right)^{a_3}\right.\\&\left.+a_4\cdot\left(\Phi_{n_{\mathrm{ch}}}^{(n)}\right)^{a_5}\cdot\left(1+a_6\cdot\left[\left|\beta_{2,\mathrm{acc}}\left(n,n_{\mathrm{ch}}\right)\right|+a_7\right]^{a_8}\right)\right\}\\\rho_{\mathrm{CUT}}^{(n)}=&\left(1+a_{23}\cdot r_{\mathrm{CUT}}^{a_{24}}\right)\cdot\left\{a_9+a_{10}\cdot\Phi_{\mathrm{CUT}}^{a_{11}}+a_{12}\cdot\Phi_{\mathrm{CUT}}^{a_{13}}\right.\\&\cdot\left(1+a_{14}\cdot R_{\mathrm{CUT}}^{a_{15}}+a_{16}\right.\\&\left.\left.\cdot\left[\left|\beta_{2,\mathrm{acc}}\left(n,n_{\mathrm{CUT}}\right)\right|+a_{17}\right]^{a_{18}}\right)\right\}\qquad(14)\end{aligned}}$$



M. Ranjbar Zefreh, F. Forghieri, S. Piciaccia, P. Poggiolini, "Accurate Closed-Form Real-Time EGN Model Formula Leveraging Machine-Learning Over 8500 Thoroughly Randomized Full C-Band Systems", J. Lightw. Technol. 38(18), p. 4987 (2020)

MITIGATION OF FIBER NONLINEARITY

NL-tolerant modulation formats

ML for NLI mitigation

Symbol-rate optimization (SRO)

Digital back-propagation (DBP)

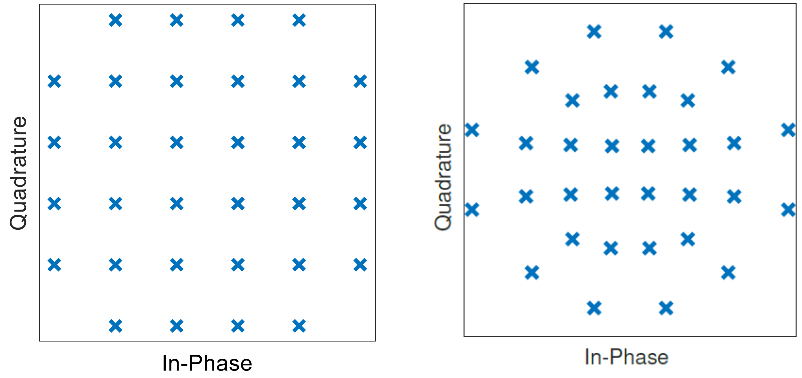
Non-linearity tolerant modulation formats

Constellation shaping

Constellation shaping

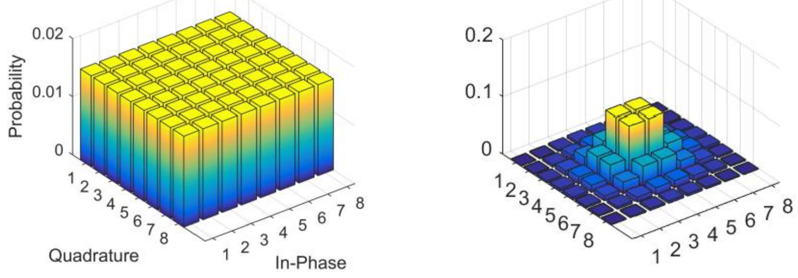
GEOMETRIC SHAPING (GS)

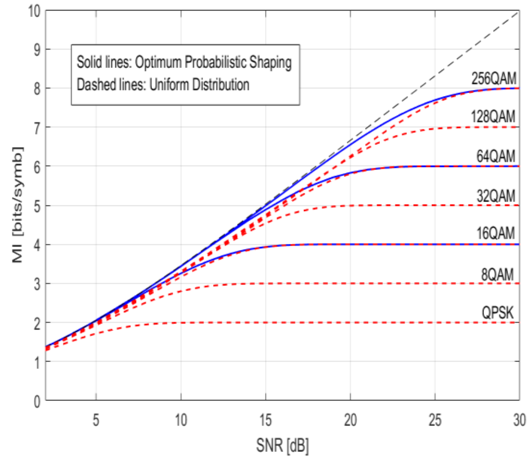
Not shaped | **Shaped**



PROBABILISTIC SHAPING (PS)

UNIFORM 64QAM | **PS-64QAM**





- Shaping can minimize the gap to the Shannon limit but typically increases the amplitude modulation, generating more Gaussian-like distributions which in turn emphasize those NLI contributions that are modulation format dependent (mainly nonlinear phase noise)
- Goal: to obtain a linear shaping gain while simultaneously keeping the amplitude modulation and the resulting nonlinear phase-noise as low as possible

NL-tolerant modulation formats

GEOMETRIC SHAPING (GS)

- Multi-dimensional ring constellations optimized for both linear and nonlinear shaping gain.
- Reduction of both the variance and the average of the transmitted signal energy.
- The overall shaping gain may exceed the 1.53 dB limit

M3A.4.pdf OFC 2016 ©

Optimized 4 and 8 Dimensional Modulation Formats for Variable Capacity in Optical Networks

M. Reimer*, S. Oveis Gharan, A. D. Shiner, and M. O'Sullivan
Ciena Corporation, 3500 Carling Ave., Ottawa, Ontario, Canada
**mreimer@ciena.com*

3884 JOURNAL OF LIGHTWAVE TECHNOLOGY, VOL. 34, NO. 16, AUGUST 15, 2016

A Shaping Algorithm for Mitigating Inter-Channel Nonlinear Phase-Noise in Nonlinear Fiber Systems

Omri Geller, Ronen Dar, Meir Feder, *Fellow, IEEE*, and Mark Shtaif, *Fellow, IEEE*

PROBABILISTIC SHAPING (PS)

- Novel low-complexity signal shaping methods which offer significant linear and nonlinear gains, as well as a good rate adaptability.

ECOC 2016
42nd European Conference and Exhibition on Optical Communications · September 18 – 22, 2016 · Düsseldorf

Low-Complexity Shaping for Enhanced Nonlinearity Tolerance

J. Cho(*), S. Chandrasekhar, R. Dar, and P. J. Winzer
Nokia Bell Labs, 791 Holmdel Road, Holmdel, NJ 07733, USA, (*) junho.cho@nokia.com

M3C.4.pdf OFC 2018

A Simple Nonlinearity-Tailored Probabilistic Shaping Distribution for Square QAM

Eric Sillekens[1], Daniel Semrau[1], Gabriele Liga[1], Nikita A. Shevchenko[1], Zhe Li[1], Alex Alvarado[2], Polina Bayvel[1], Robert. I. Killey[1], and Domaniç Lavery[1]
1) Optical Networks Group, Dept. of Electronic & Electrical Engineering, University College London, UK
2) Signal Processing Systems Group, Dept. of Electrical Engineering, Eindhoven University of Technology (TU/e), NL
e.sillekens@ucl.ac.uk

Symbol rate optimization (SRO)

UNIFORM LINKS, IDEAL NYQUIST-WDM

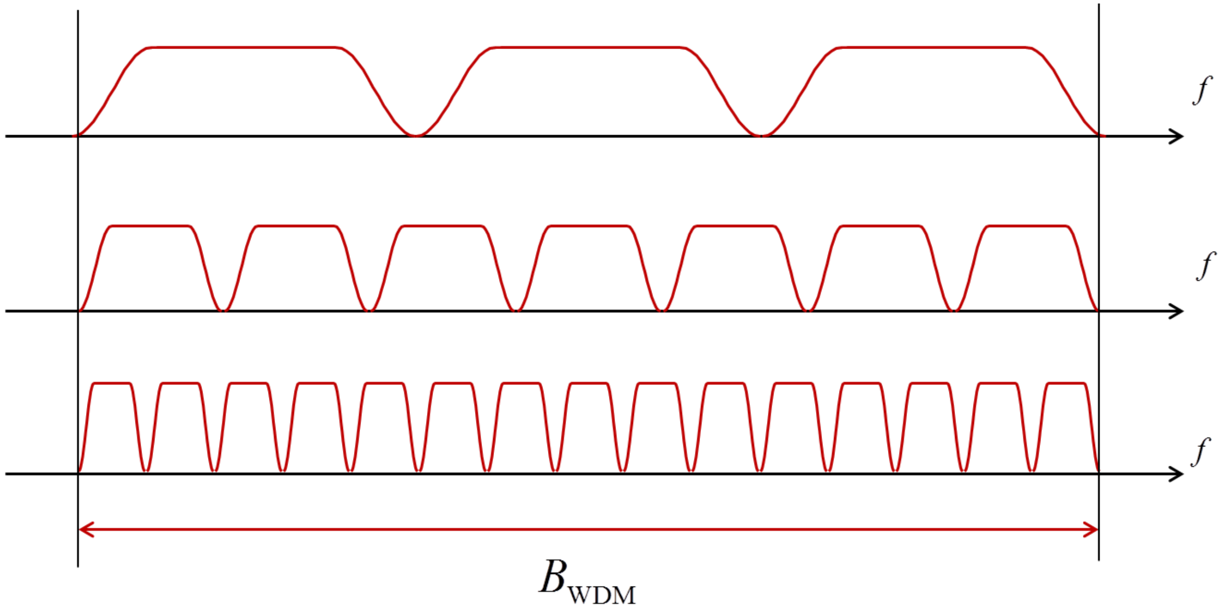
$$\boxed{R_{opt} = \sqrt{\frac{2}{\pi\left|\beta_2\right| L_s N_s}}}$$

P. Poggiolini et al., "Analytical and experimental results on system maximum reach increase through symbol rate optimization," J. Lightw. Technol., 34(8), p. 1872 (2016).

An example

- **What is the symbol rate which minimizes NLI ?...**
  …having fixed:
  - the total WDM bandwidth (B_{WDM}=500 GHz, 5 THz)
  - the modulation format and roll-off (PM-QPSK, ρ=0.05)
  - the relative frequency spacing (Δf=1.05 R_s)

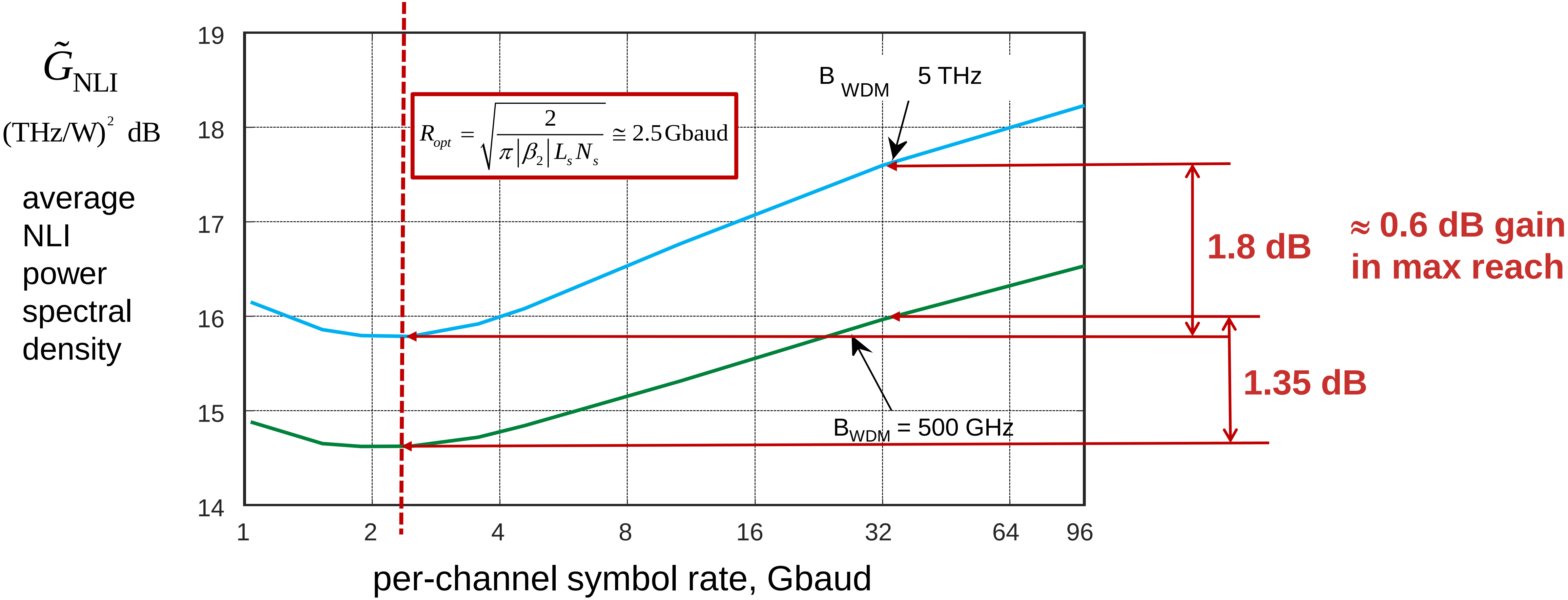
- EDFA-only amplification (F=5 dB)
- SSMF fiber (100-km span length)



P. Poggiolini et al., "Analytical and experimental results on system maximum reach increase through symbol rate optimization," J. Lightw. Technol., 34(8), p. 1872 (2016).

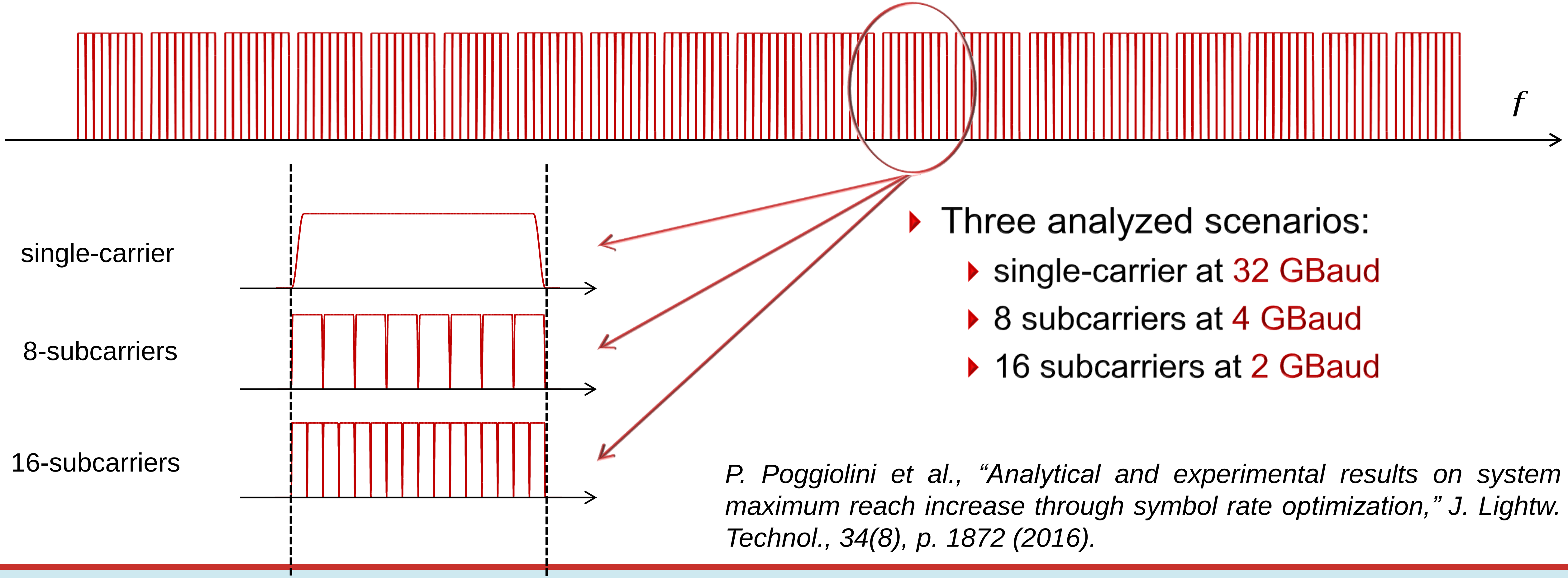
SRO prediction by EGN model

- **PM-QPSK,** roll-off 0.05, spacing 1.05 x (symb rate), **SMF, 100 km spans, 50 spans**



SRO through sub-carrier multiplexing

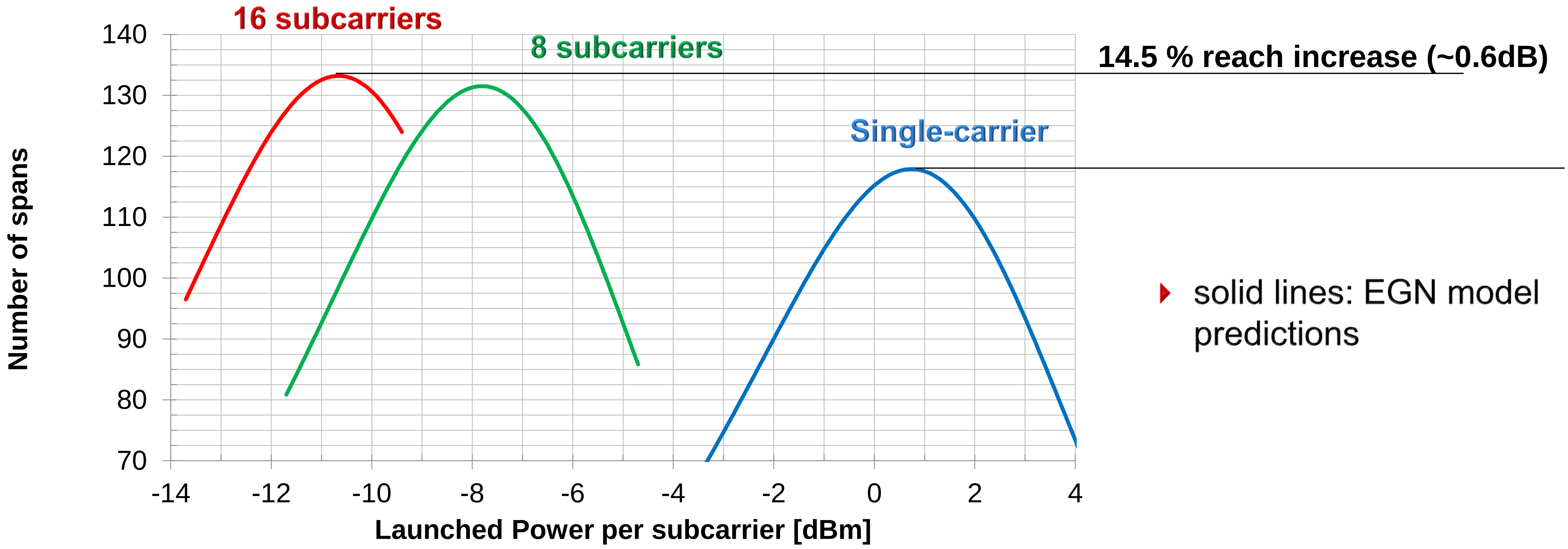
- **19 channel** WDM comb, with channel **spacing 37.5 GHz**, for a total WDM bandwidth of **710 GHz**



- Three analyzed scenarios:
  - single-carrier at 32 GBaud
  - 8 subcarriers at 4 GBaud
  - 16 subcarriers at 2 GBaud

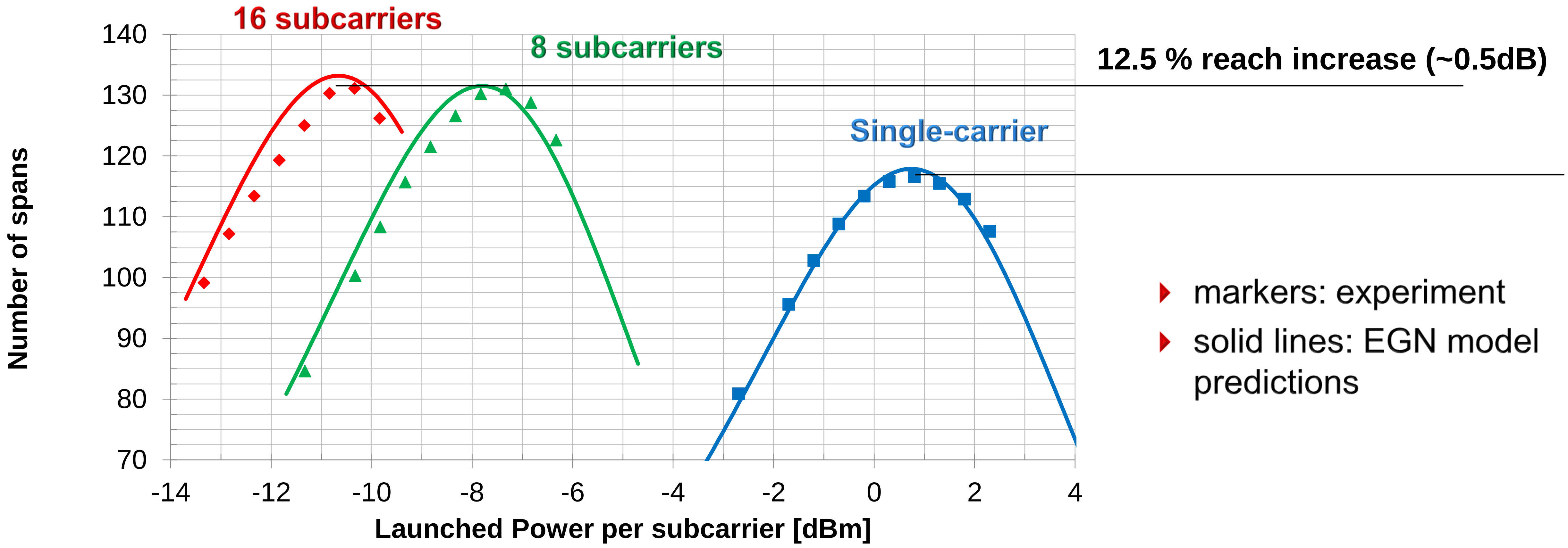
P. Poggiolini et al., "Analytical and experimental results on system maximum reach increase through symbol rate optimization," J. Lightw. Technol., 34(8), p. 1872 (2016).

Reach curves over PSCF fiber (108 km spans)



- solid lines: EGN model predictions

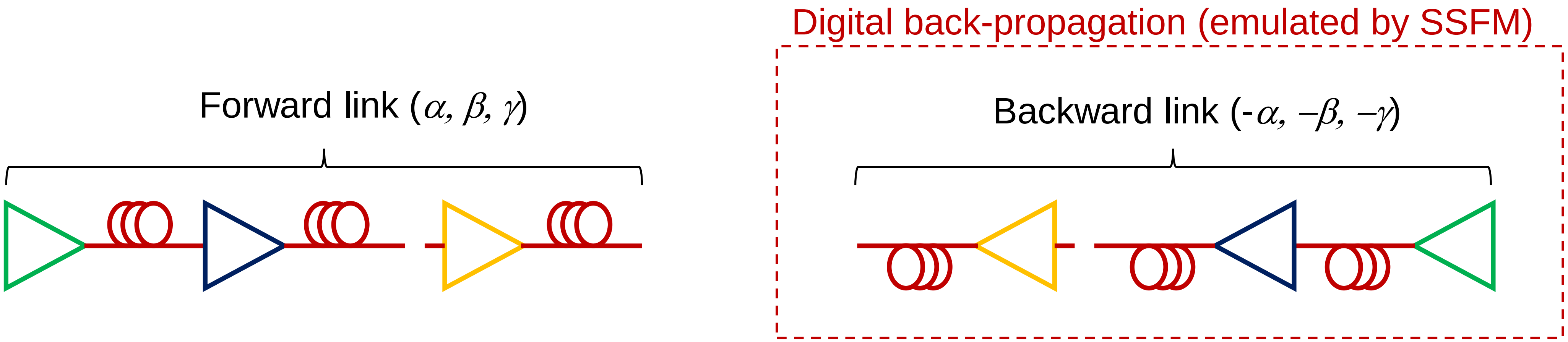
Reach curves over PSCF fiber (108 km spans)



- markers: experiment
- solid lines: EGN model predictions

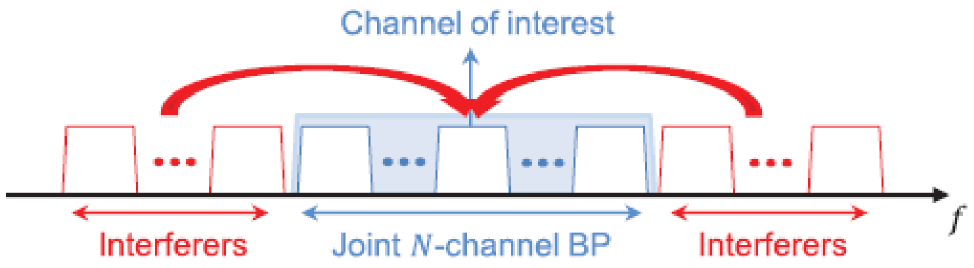
- The gain predicted by the analytical model cannot be fully exploited due to practical implementation issues (higher sensitivity to transceiver impairments and phase noise)

Digital back-propagation (DBP)

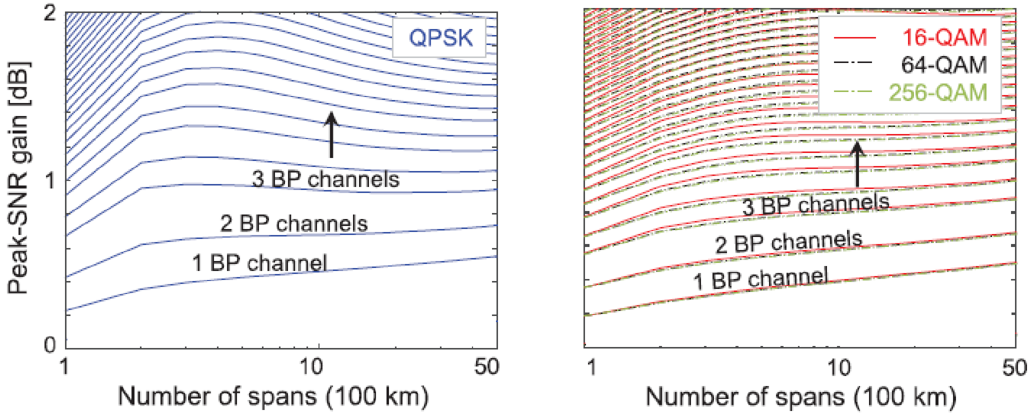


- Ideal performance if:
  - Full-bandwidth
  - High number of steps per span

Theoretical performance

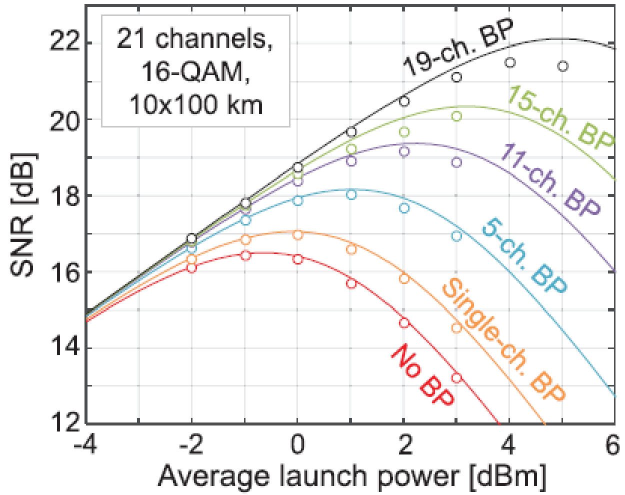


Fully loaded system with 115 channels at 32 Gbaud



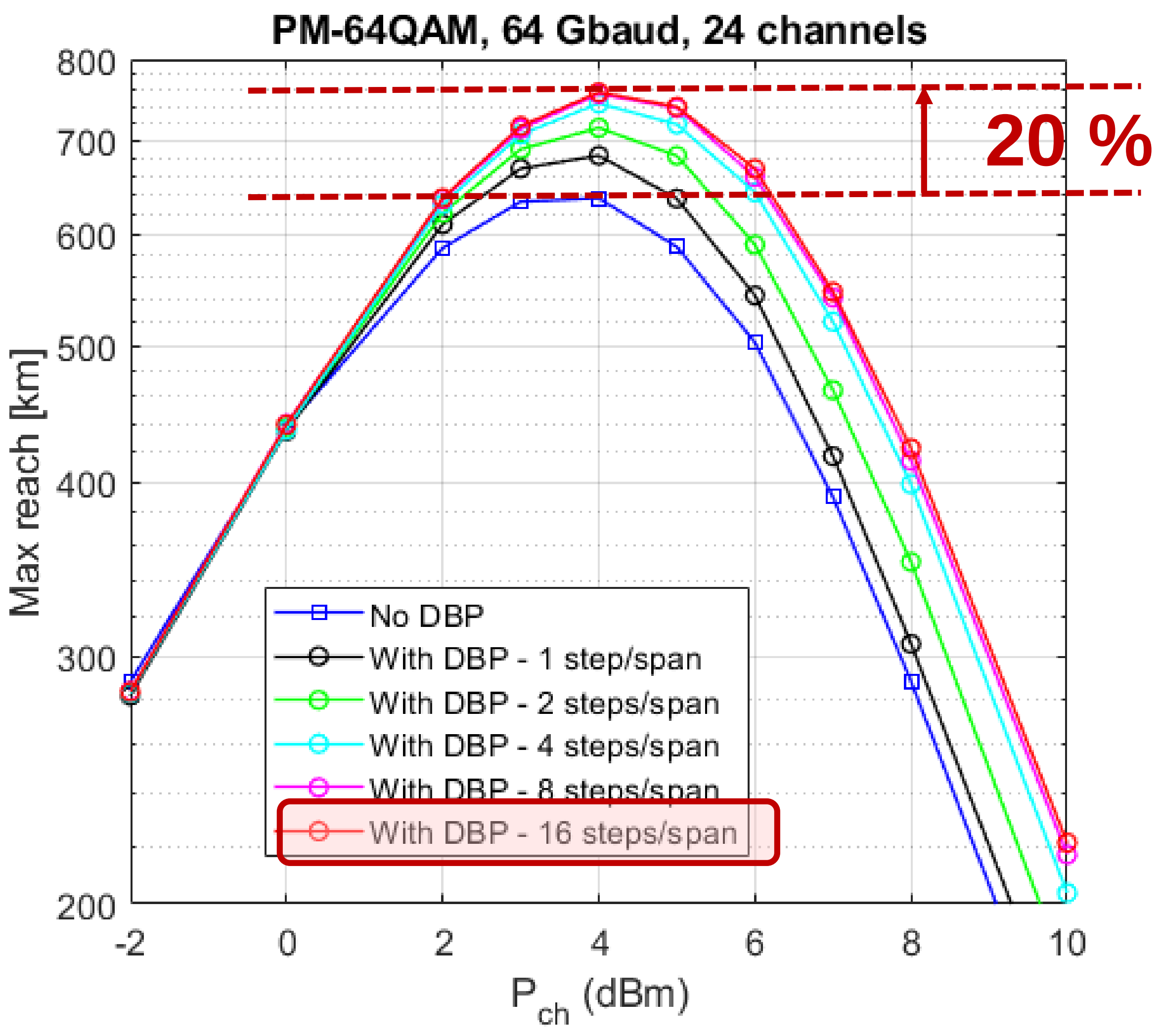
Ideal gains

Including ASE noise induced nonlinearities



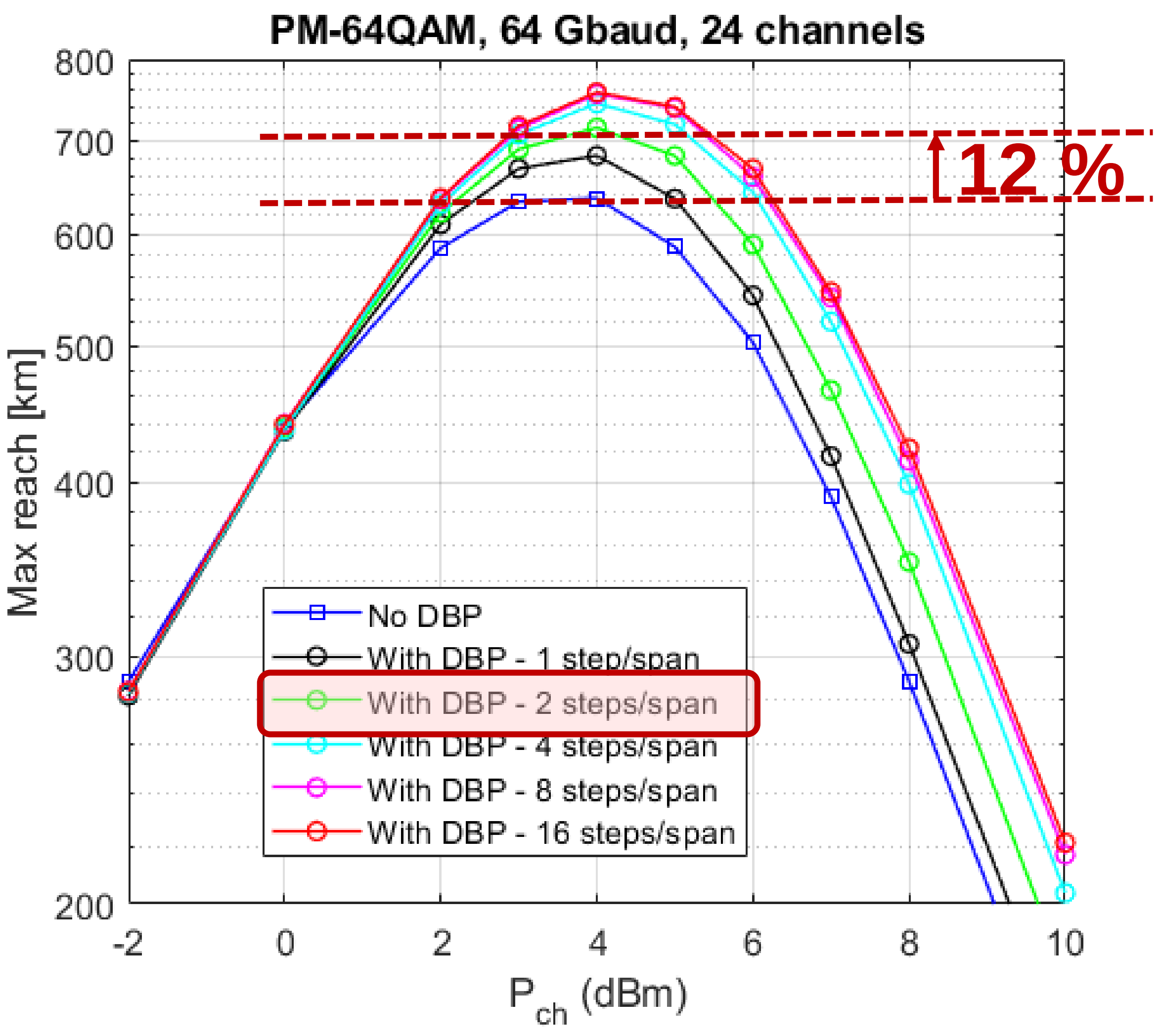
R. Dar and P. Winzer, "Nonlinear Interference Mitigation: Methods and Potential Gain," J. Lightw. Technol. 35(4), p. 903 (2017).

DBP performance vs. number of steps per span



- Modulation format: PM-64QAM
- Roll-off: 0.2
- SSMF fiber - 100 km spans
- EDFA noise figure: 6 dB
- Target GMI: 5.22 bit/symb → Target SNR: 17.37 dB
- Channel spacing: 1.2 R_s = 76.8 GHz
- Single-channel DBP

DBP performance vs. number of steps per span



- Modulation format: PM-64QAM
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- EDFA noise figure: 6 dB
- Target GMI: 5.22 bit/symb → Target SNR: 17.37 dB
- Channel spacing: 1.2 R_s = 76.8 GHz
- Single-channel DBP

Key take-aways, so far …

- The **NLI analytical models** are useful tools to obtain an accurate prediction of the **ultimate performance** achievable by the various mitigation techniques.

- The **actual performance gain** will also depend on several implementation issues that cannot be easily included in the analytical estimations, such as:
  - sub-optimum performance of low-complexity DBP algorithms
  - higher impact of NLPN in digital multi-subcarrier systems

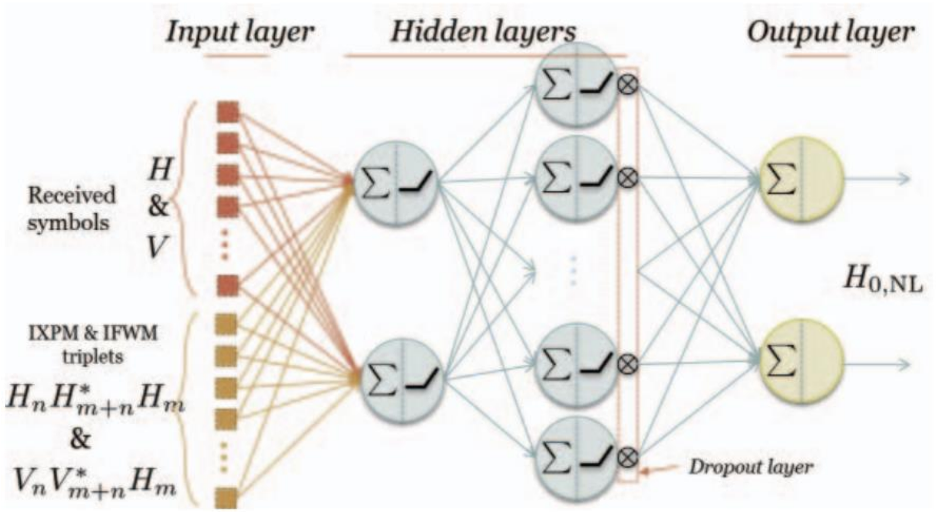
  which reduce the nonlinearity mitigation benefits.

Machine-learning (ML) for NLI mitigation

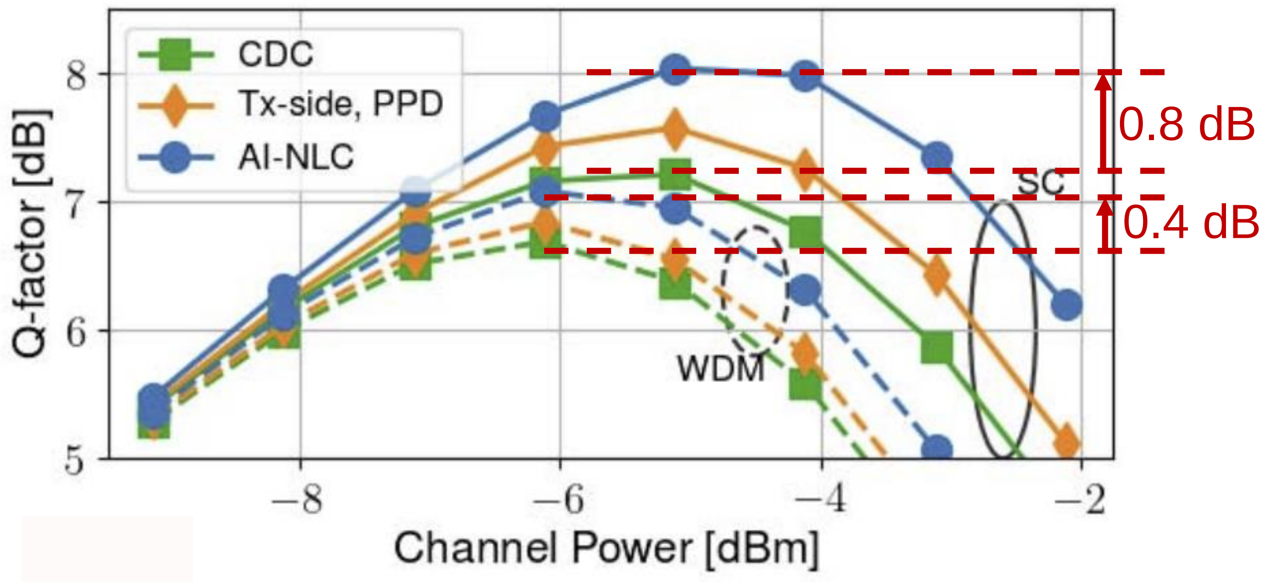
The received symbols are treated as ordinary data samples and develop a ML model for symbol detection without considering system parameters

Fiber parameters are integrated into ML modeling, thus using more comprehensive knowledge of optical fibers and transmission systems

System-agnostic Deep-Neural Networks (DNN)



4x12.5 Gbaud PS-64QAM

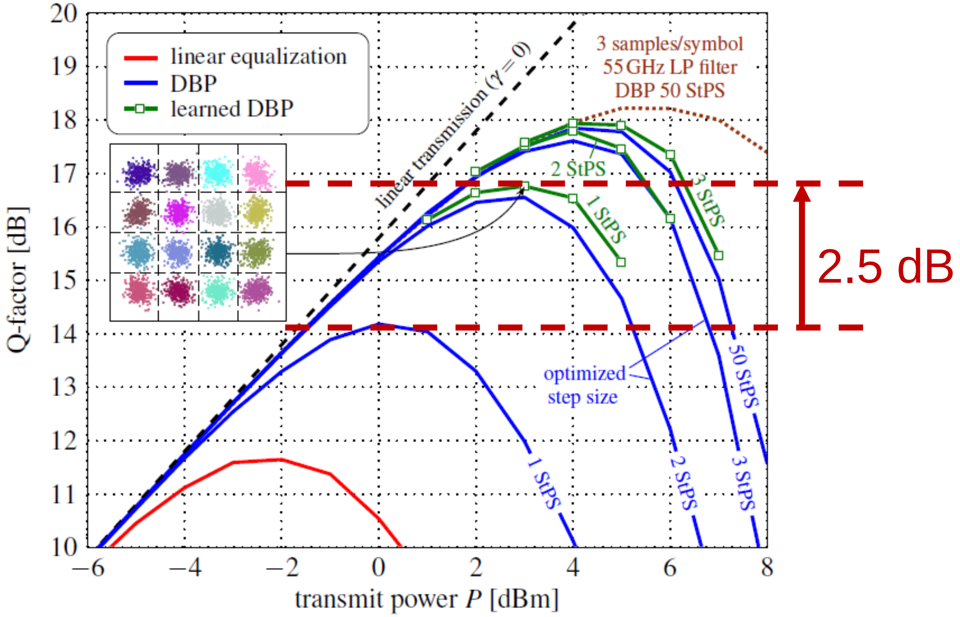


V. Kamalov et al., "Evolution from 8qam live traffic to ps 64-qam with neural network based nonlinearity compensation on 11000 km open subsea cable", OFC 2018, *PDP Th4D.5, San Diego, CA*

Learned -DBP

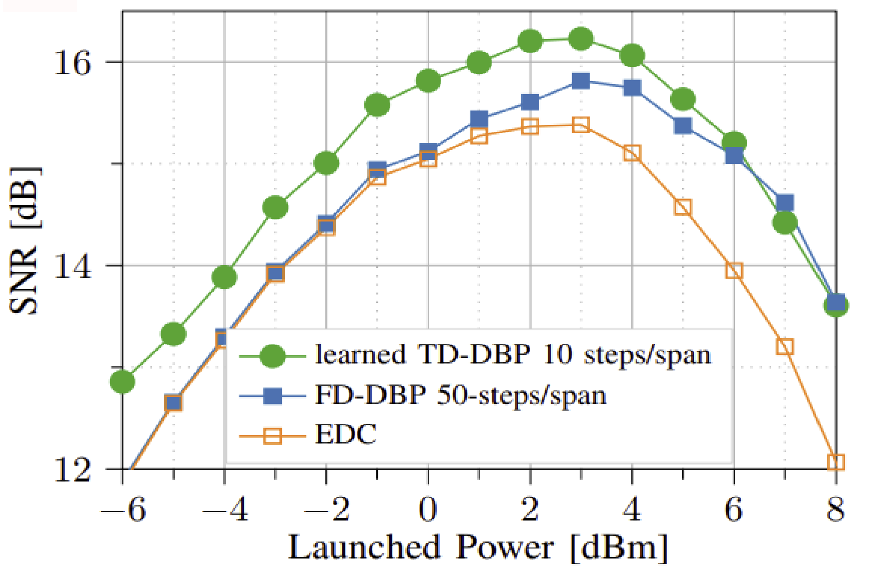
- DBP has a similar mathematical structure as a neural network

Simulation results
Single-channel 20-GBd PM-16QAM signal transmission over 32 spans of SMF



C. Häger and H. D. Pfister, "Nonlinear interference mitigation via deep neural networks," OFC 2018

Experimental demonstration
Four channel 64-GBd PM-64QAM signal transmission over 10 spans of SMF



E. Sillekens et al., "Time-Domain Learned Digital Back-Propagation," 2020 IEEE Workshop on Signal Proc. Systems.

Nonlinearity mitigation at ECOC 2020

- WS9: Bin Chen (Hefei Univ. of Technol.): "Multi-dimensional geometric shaping for high-capacity nonlinearity-tolerant transmission"
- Mo2E-2: S. Beppu (KDDI Research) et al., "Verification on Digital Back Propagation Gain in MCF transmission over 6020-km Uncoupled and Coupled 4-Core Fibres"
- Tu1F-4: P.M. Kaminski (DTU Fotonik) et al., "All-Optical Nonlinear Pre-Compensation of Long-Reach Unrepeatered Systems"
- Tu2F-7: Junho Cho, Xi Chen (Nokia Bell Labs), "On Small Multi-Dimensional Constellations for Nonlinear Optical Fiber Communications"
- We1D-2: P.J. Freire (Aston Univ.) et al., "Experimental Verification of Complex-Valued Artificial Neural Network for Nonlinear Equalization in Coherent Optical Communication Systems"
- We1D-4: S. Deligiannidis (Univ. of West Attica) et al. "Performance and Complexity Evaluation of Recurrent Neural Network Models for Fibre Nonlinear Equalization in Digital Coherent Systems"
- We1E-8: J. Koch (Kiel Univ.) et al., "Neural Networks based Equalization of Experimental Transmission using the Nonlinear Fourier Transformation"
- We1F-2: B. Chen (Hefei Univ. of Technol.) et al., "Nonlinear Interference Analysis of Probabilistic Shaping vs. 4D Geometrically Shaped Formats"
- We1F-3: S. Civelli (Scuola Superiore Sant'Anna) et al. "Interplay of Probabilistic Shaping and Carrier Phase Recovery for Nonlinearity Mitigation"
- Th1D-5: V. Neskorniuk (Aston University) et al., "Simplifying the Supervised Learning of Kerr Nonlinearity Compensation Algorithms by Data Augmentation"
- Th2G-5: Li Zhao (Fudan Univ.) et al., "Demonstration of 73.15Gbit/s 4096-QAM OFDM D-band Wireless Transmission Employing Probabilistic Shaping and Volterra Nonlinearity Compensation"

POLITECNICO DI TORINO

Advances in Modeling and Mitigation of Nonlinear Effects in Uncompensated Coherent Optical Transmission Systems

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