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# Digital Nonlinear Equalization

## for Long-Haul Optical Transmission Systems

Fernando P. Guiomar

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Instituto de Telecomunicações - Pólo de Aveiro, Portugal*

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- Other things I like to do:
  - Football, music, cinema, travelling...

- Summary of my PhD Thesis Work;
  - Review of Digital Backpropagation;
  - Frequency Domain Volterra Series Nonlinear Equalizer;
  - Time Domain Volterra Series Nonlinear Equalizer;
  - Multi-Carrier Digital Backpropagation.
- Some Open Topics on Nonlinear Equalization;
- The FLEX-ON Project;
  - Main Scientific Objectives;
  - Dissemination and Public Engagement.

# **Review of Digital Backpropagation**

# Digital Backpropagation - Operation Principle

- Effective linear+nonlinear compensation requires channel inversion techniques;
  - Signal propagation in the direct fiber direction can be described by:

$$\frac{\partial A_{x/y}}{\partial z} = -\frac{\alpha}{2} A_{x/y} - i \frac{\beta_2}{2} \frac{\partial^2 A_{x/y}}{\partial^2 t} + i \frac{8}{9} \gamma (|A_x|^2 + |A_y|^2) A_{x/y},$$

# Digital Backpropagation - Operation Principle

- Effective linear+nonlinear compensation requires channel inversion techniques;
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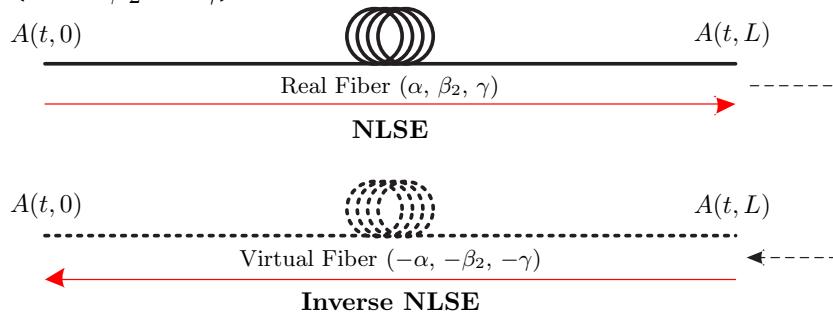
$$-\frac{\partial A_{x/y}}{\partial z} = +\frac{\alpha}{2}A_{x/y} + i\frac{\beta_2}{2}\frac{\partial^2 A_{x/y}}{\partial^2 t} - i\frac{8}{9}\gamma(|A_x|^2 + |A_y|^2)A_{x/y},$$

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- This corresponds to propagate the received signal through a **virtual fiber** with parameters with the opposite sign ( $-\alpha, -\beta_2$  e  $-\gamma$ ).

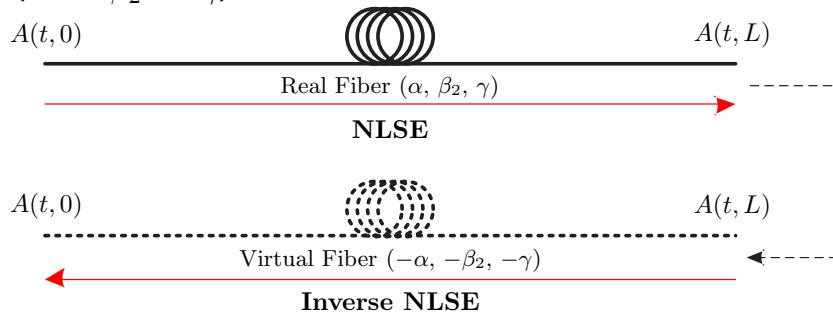


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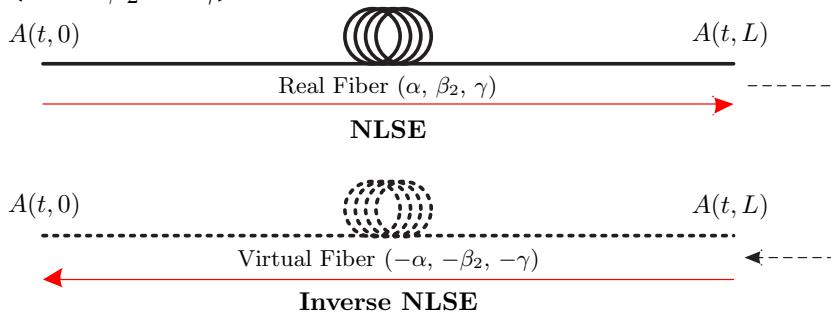
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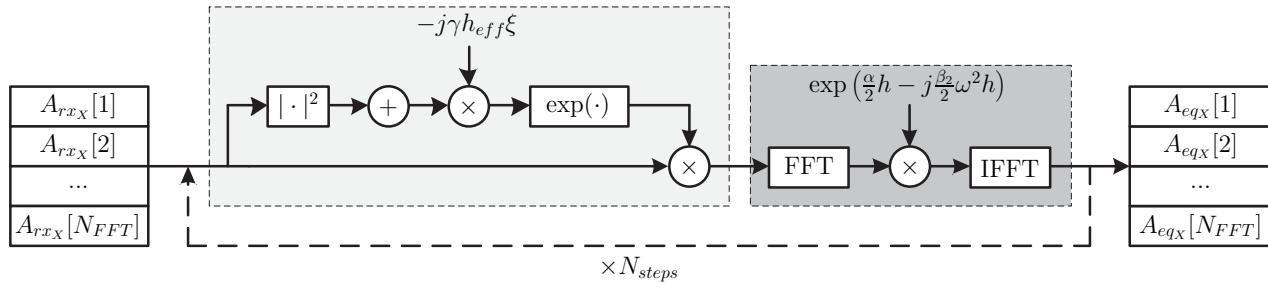
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- This method can be digitally applied at the receiver-side for **universal impairment compensation**;
- Equalization performance and computational efficiency strongly depend on the numerical method used to apply DBP.

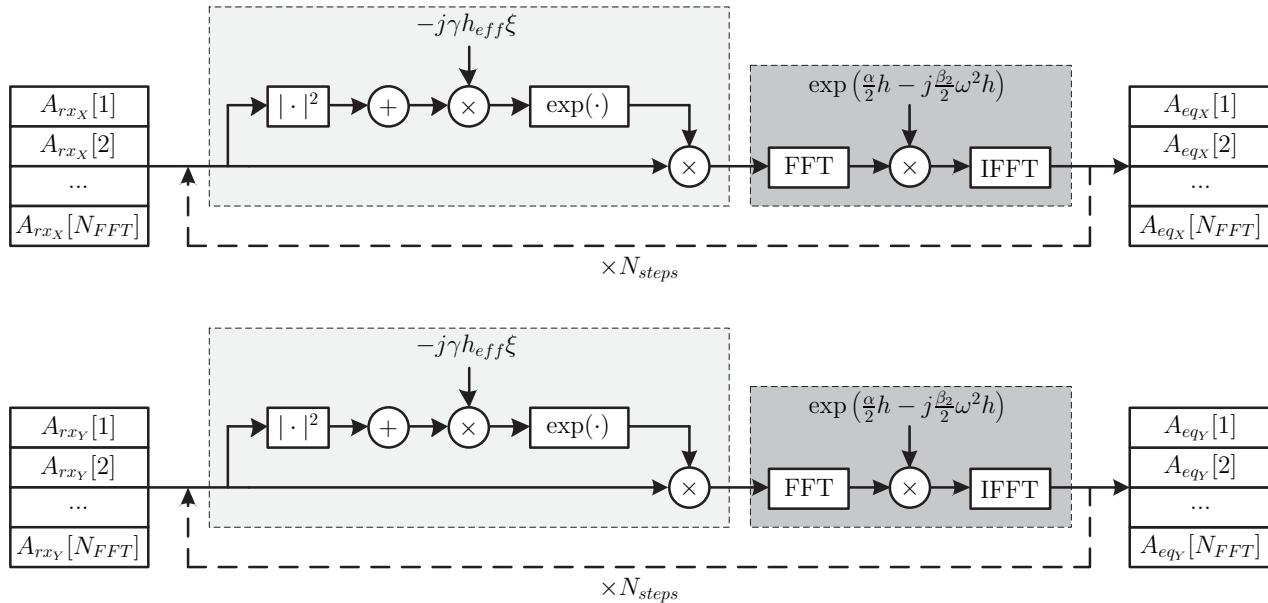
# Backpropagation Split-Step Fourier Method

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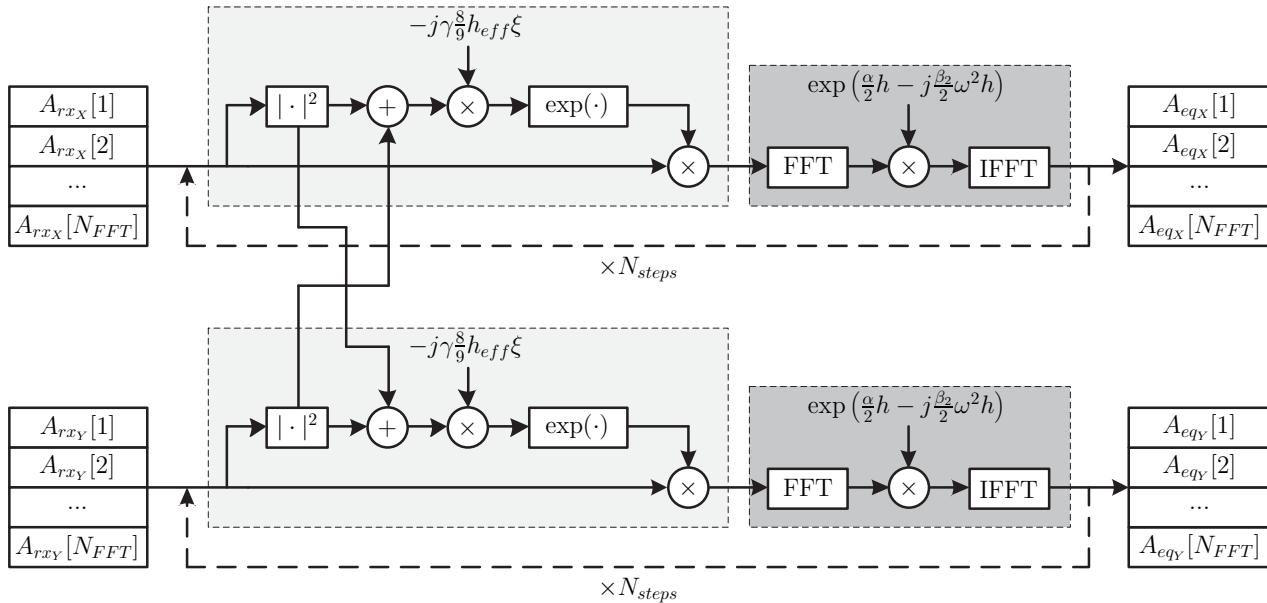
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# Backpropagation Split-Step Fourier Method

- BP-SSFM has been the most widely used numerical method to solve DBP;



- If spatial and temporal resolutions are sufficiently high, BP-SSFM enables to **fully compensate all the deterministic impairments**.

# Inverse Volterra Series Transfer Function

## 3.1) Analytical Description

# VSTF - Analytical Description

- **DBP alternative:** third-order Volterra series expansion of the inverse NLSE in frequency-domain:

$$\tilde{A}_{x/y}(\omega, z-L_s) \approx \underbrace{K_1(\omega, L_s)\tilde{A}_{x/y}(\omega, z)}_{\text{linear term}} + \underbrace{\Gamma(\omega, L_s) \iint K_3(\omega, \omega_j, \omega_k) \tilde{P}(\omega_j, \omega_k, z) \tilde{A}_{x/y}(\omega + \omega_j - \omega_k, z) \partial\omega_j \partial\omega_k}_{\text{3rd-order nonlinear term}}$$

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where  $\tilde{P}(\omega_j, \omega_k, z)$  includes the inter-polarization crosstalk,

$$\tilde{P}(\omega_j, \omega_k, z) = \tilde{A}_x(\omega_k, z) \tilde{A}_x^*(\omega_j, z) + \tilde{A}_y(\omega_k, z) \tilde{A}_y^*(\omega_j, z),$$

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$K_3(\omega, \omega_j, \omega_k)$  is the inverse nonlinear kernel,

$$K_3(\omega, \omega_j, \omega_k) = \frac{1 - \exp\left(\alpha L_s - i\beta_2(\omega_k - \omega)(\omega_k - \omega_j)L_s\right)}{-\alpha + i\beta_2(\omega_k - \omega)(\omega_k - \omega_j)},$$

and  $\Gamma(\omega, z)$  is a one-dimensional frequency-dependent nonlinear term,

$$\Gamma(\omega, z) = -i\xi\frac{8}{9}\gamma K_1(\omega, z).$$

# VSNE - 2D Representation

- The Volterra series nonlinear equalizer in frequency domain is based on an entry-wise product of  $N \times N$  matrices:

$$\tilde{\mathbf{N}}_{\mathbf{x}/\mathbf{y}}(\omega_n, z) = \mathbf{K}_3(\omega_n) \circ \tilde{\mathbf{P}}(z) \circ \tilde{\mathbf{A}}_{\mathbf{x}/\mathbf{y}}(\omega_n, z)$$

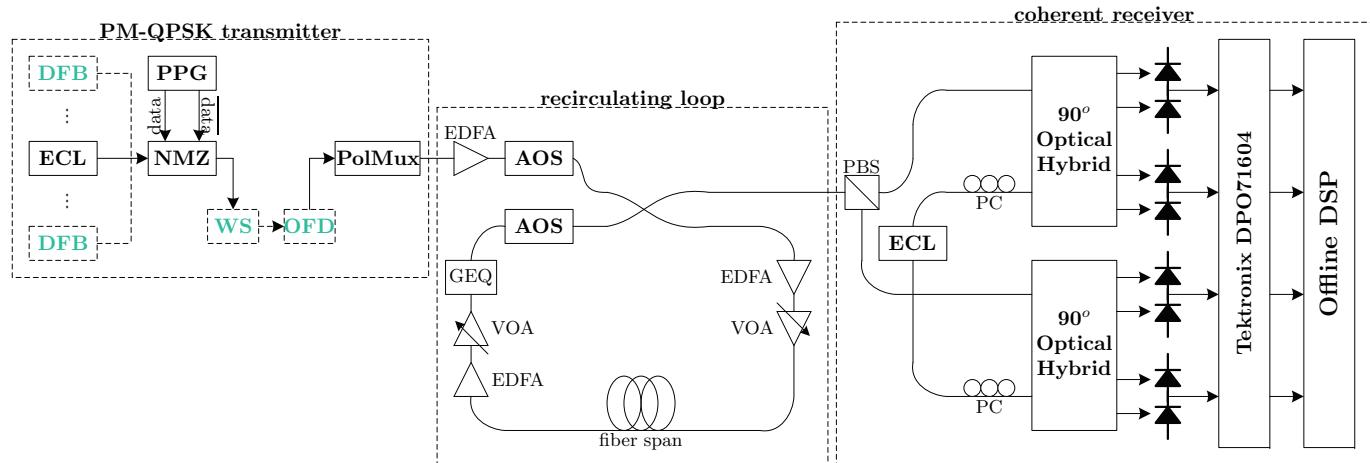
- Highly parallel** implementation, but with  $O(N^2)$  complexity per processed sample.

# **Inverse Volterra Series Transfer Function**

## **3.2) Experimental Results**

# VSNE - Experimental Setup

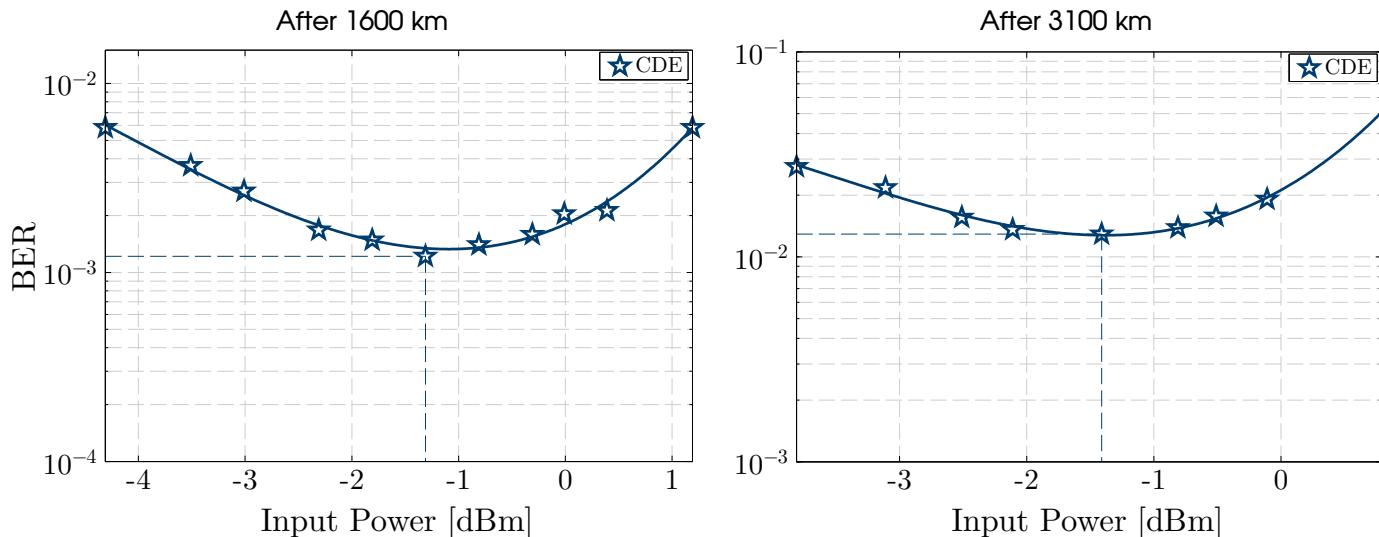
- Experimental setup of a **100G PM-QPSK experiment**, carried out in collaboration with Politecnico di Torino, Italy, in the framework of the EURO-FOS project:



- Main features:
  - Single-channel PM-QPSK @ 120 Gb/s;
  - Optical Nyquist pulse shaping;
  - Propagation over NZDSF spans with 100 km each;
  - Sampling @ 50 Gsa/s -  $\sim 1.6$  samples per symbol.

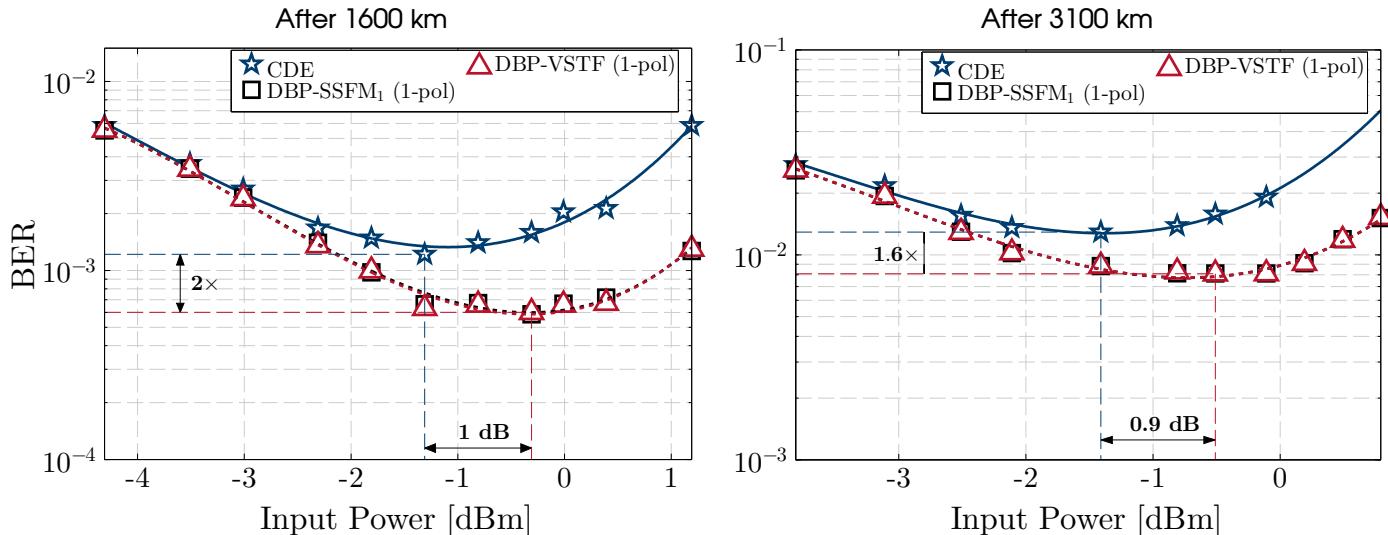
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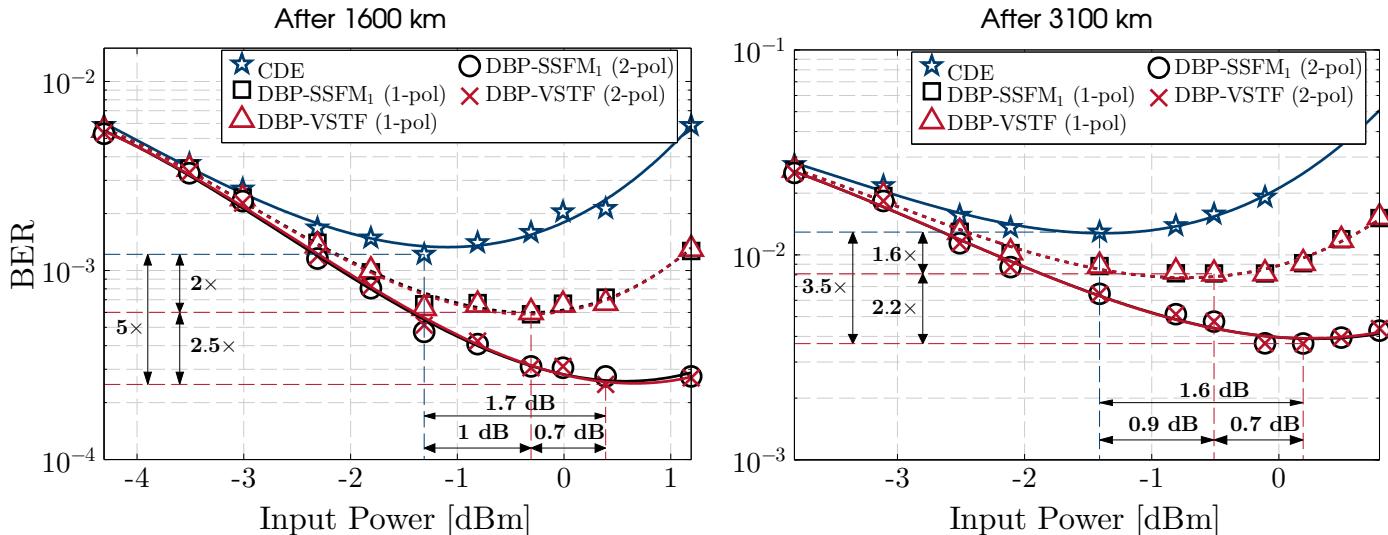
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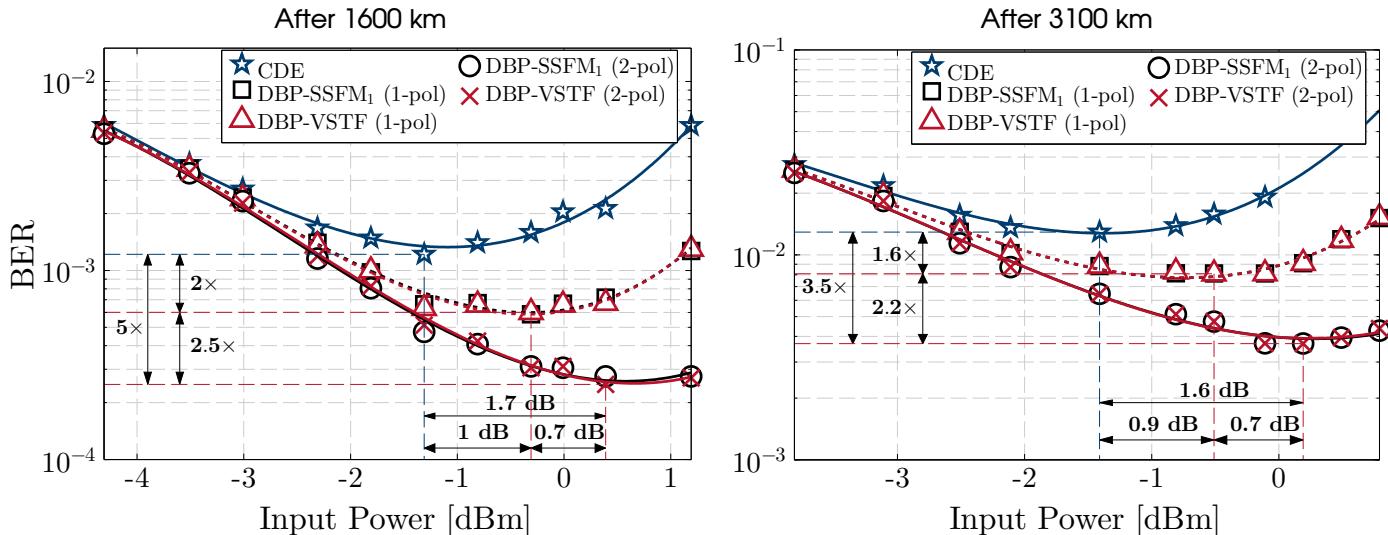
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- The single-polarization DBP-SSFM and DBP-VSTF increase the nonlinear tolerance by  $\sim 1$  dB;
- Another 0.7 dB is obtained by the dual-polarization models, with negligible added complexity;
- The DBP-SSFM and DBP-VSTF are shown to yield similar accuracy.

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- But, can we reduce the complexity of the DBP-VSTF?

# Frequency Domain VSNE

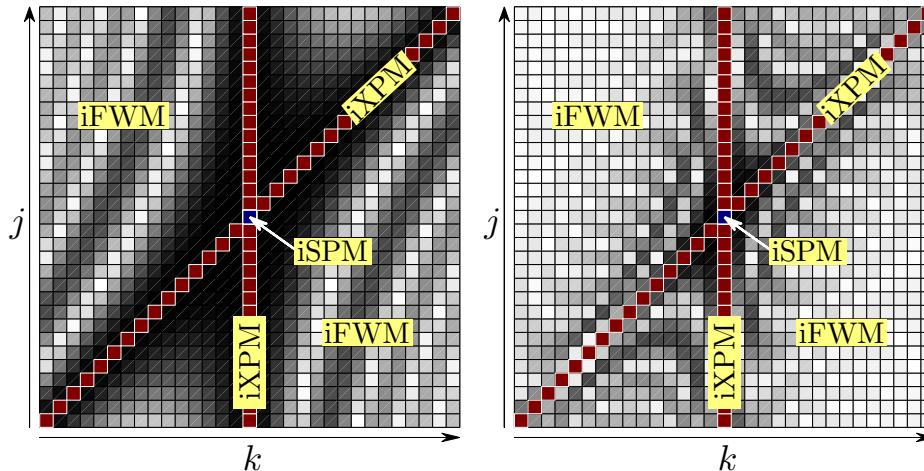
## 4.1) Analytical Description

# VSNE - 3<sup>rd</sup> Order Kernel

**NZDSF:**

$N = 32; n = 16$

**SSMF:**



$iSPM \rightarrow j = k = n;$   
 $iXPM \rightarrow j = k \neq n;$   
     $\rightarrow k = n \neq j;$   
 $iFWM \rightarrow \text{otherwise}$

- Interesting **symmetries** can be exploited to **avoid replication** of operations;
  - Some of the symmetries are also shared with the  $\tilde{A}_{x/y}$  matrices.
- The  $K_3$  pattern strongly **depends on the accumulated dispersion**;
- The real part of  $K_3$  is **maximum at the iXPM+iSPM contribution**.

# VSNE - Symmetric Reconstruction

- Starting from the iXPM contribution (main diagonal +  $n$ -th column), the  $K_3$  kernel can be reconstructed from its symmetric column/diagonal pairs;
- The full VSNE kernel is then **decomposed into a set of  $N_K$  parallel frequency-domain filters**:
  - symVSNE**: complexity is reduced from  $O(N^2)$  to  $O(N_k N)$ ;
  - simVSNE**: constant-coefficient approximation, **numerical complexity is  $O(N_k)$** .

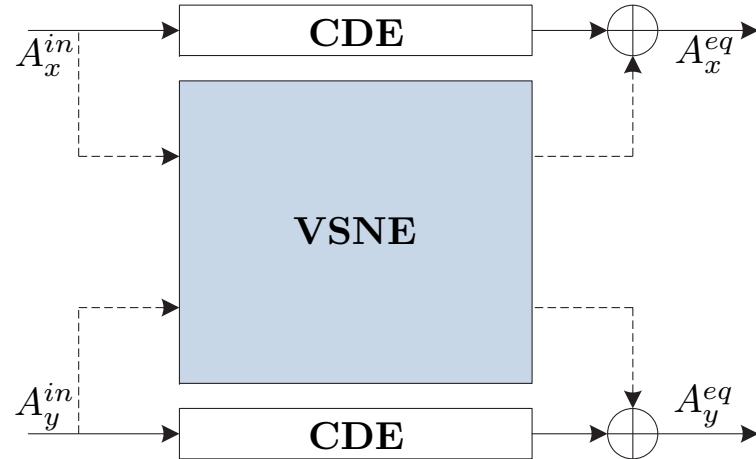
# Backpropagation using VSNE-based Equalizers

- DBP implementation using CDE and VSNE for linear and nonlinear compensation:



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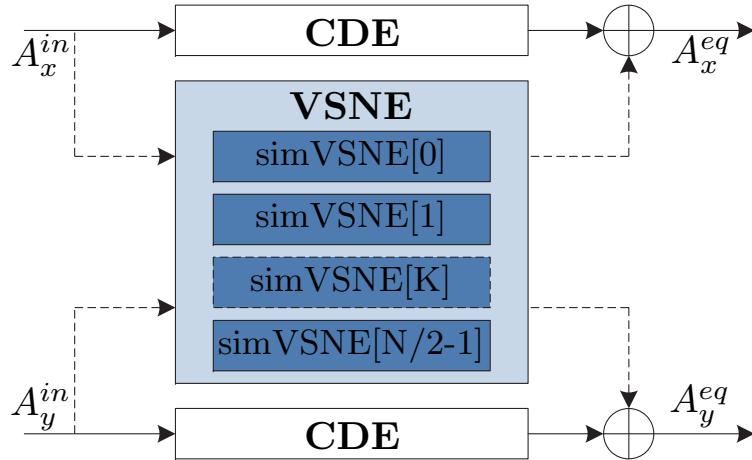
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- The VSNE module is **applied in parallel** with CDE;
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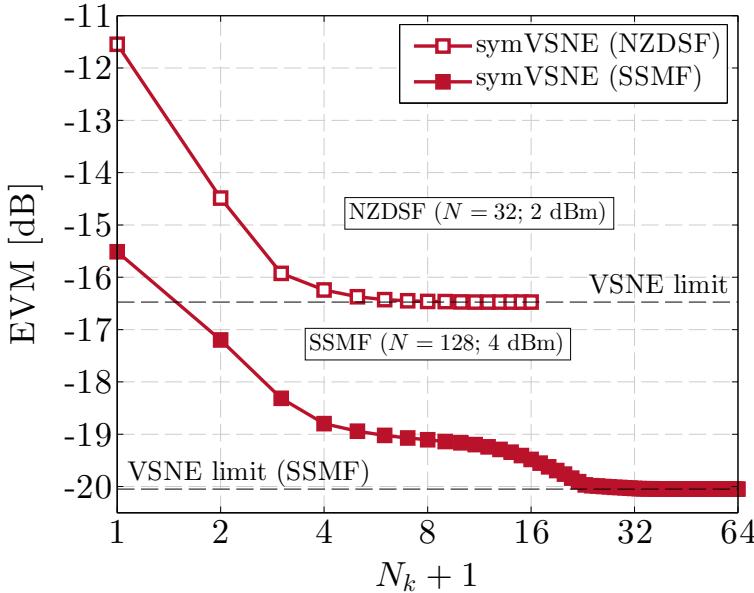
- The VSNE module is **applied in parallel** with CDE;
  - The performance is improved by perturbing the CDE output with a **nonlinear error signal**.
- The VSNE can be subdivided into a **set of  $N_k$  parallel simVSNE equalizers**.
  - Nonlinear equalization becomes an **highly granular add-on**;
  - Easier trade-off **between performance and complexity**, depending on the **DSP resources**.

# Frequency Domain VSNE

## 4.2) Simulation and Experimental Results

# simVSNE - Filter Dimension and Computational Effort

- Simulation results for a  $20 \times 80$  km 224 Gb/s PM-16QAM transmission system:

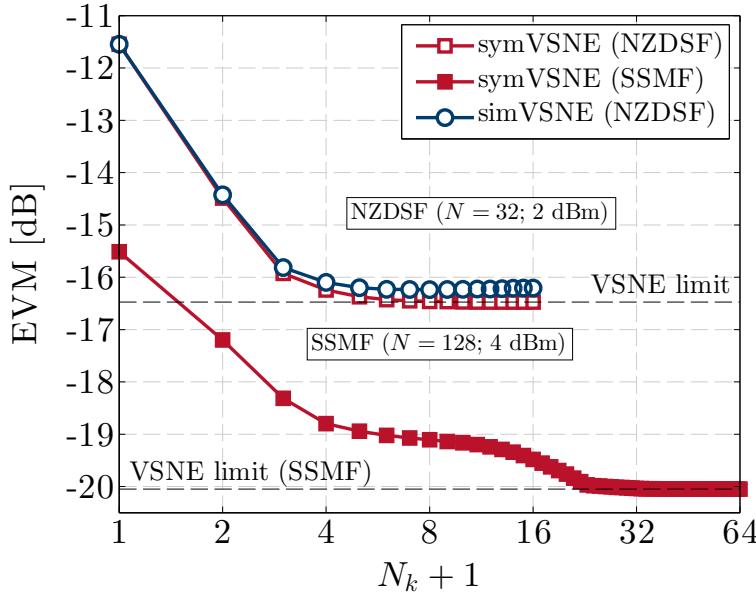


**Number of CMs per sample:**

$N_k$	symVSNE		simVSNE	
	$N = 32$	$N = 128$	$N = 32$	$N = 128$
1	6	6	6	6
2	134.9	519	17.6	17.9
3	247.6	1015.2	28.9	29.7
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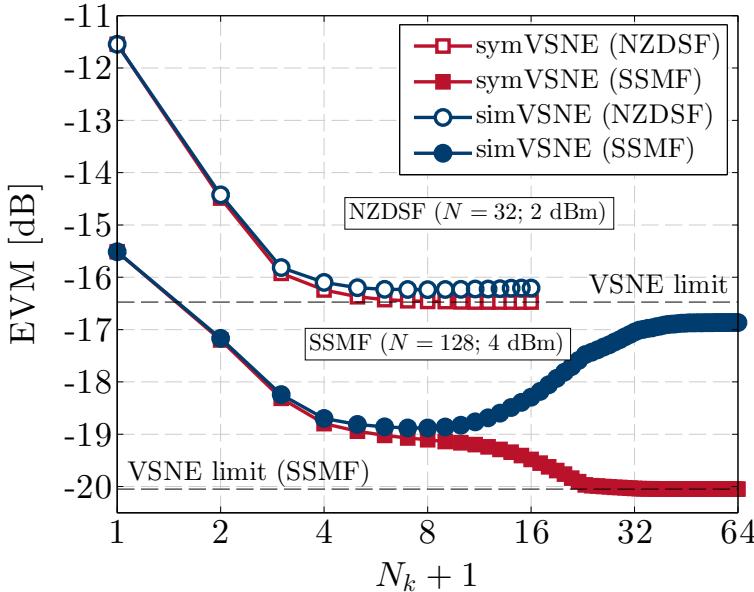
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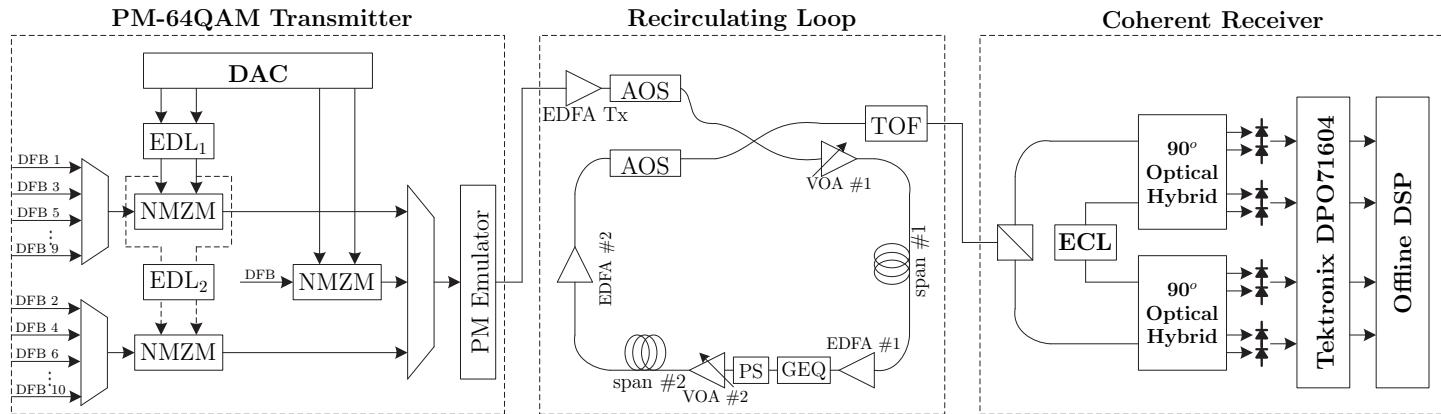
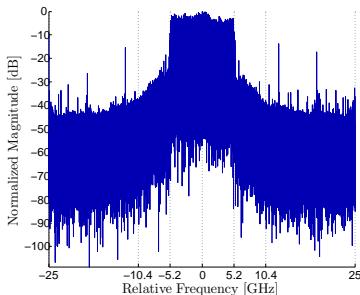
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- The simVSNE error is negligible for low-dispersion fibers;
- With large  $\beta_2$  the iXPM-like regions in the VSNE kernel tend to become **narrower**;
- Still, **for small  $N_k$** , the **constant-coefficient assumption remains valid!**

# Laboratorial Setup

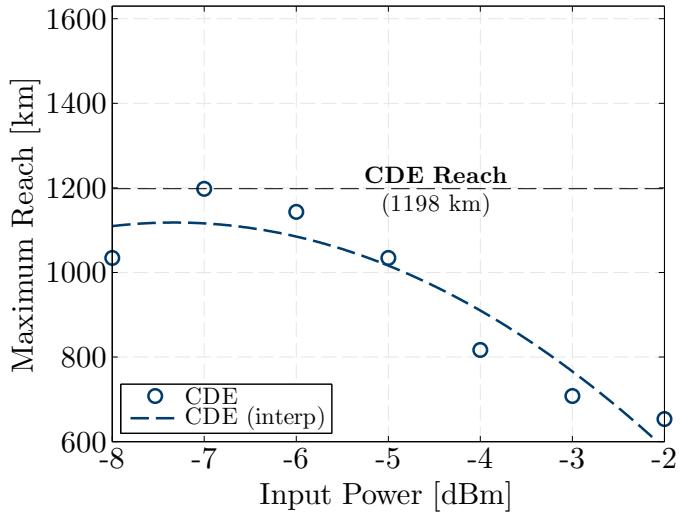
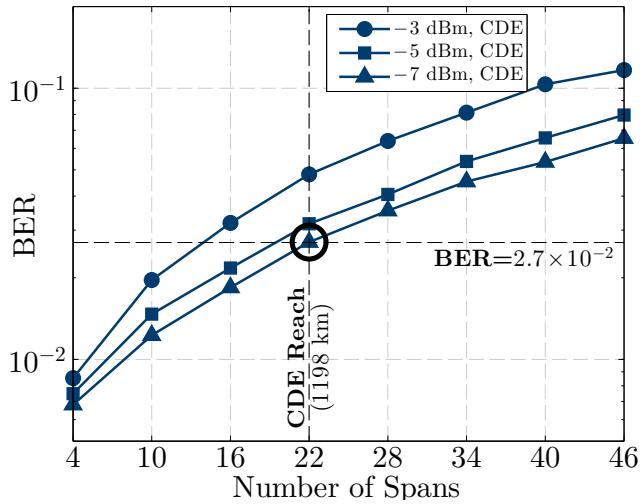
- **100G PM-64QAM transmission system** implemented in collaboration **Politecnico di Torino** and **Istituto Superiore Mario Boella**, Italy:

- **124.8 Gb/s PM-64QAM** signal;
- Recirculating loop composed of  $2 \times 54.44$  km of PSCF ( $150 \mu\text{m}^2$ );
- **10 WDM channels** spaced by 50 GHz;
- Digital Nyquist pulse shaping.



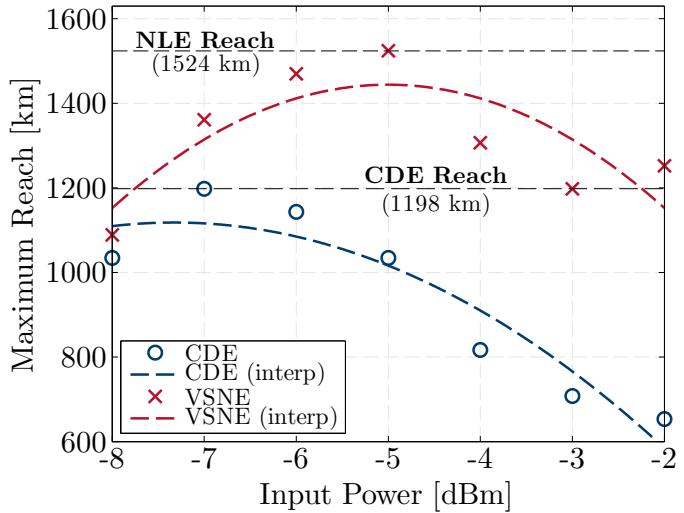
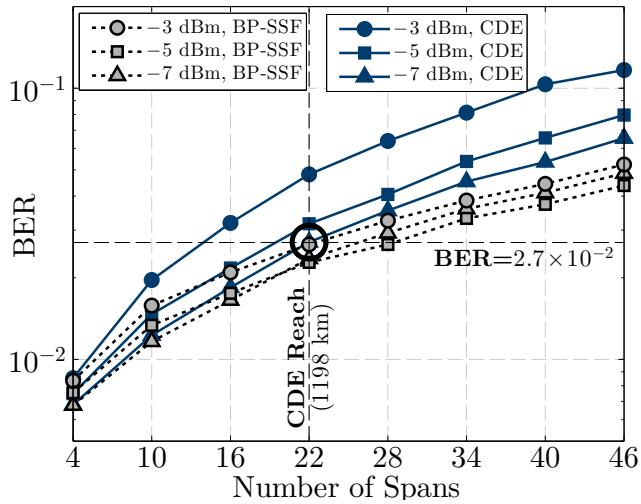
# simVSNE - Experimental Demonstration

- BER performance of 124.8 Gb/s PM-64QAM after linear and nonlinear equalization:



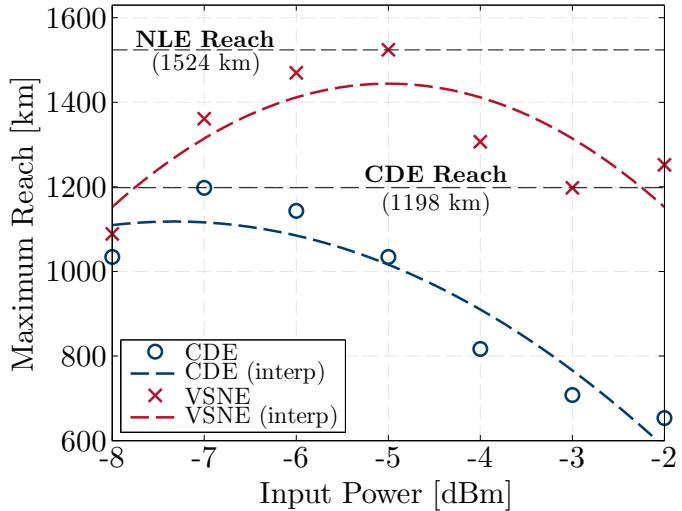
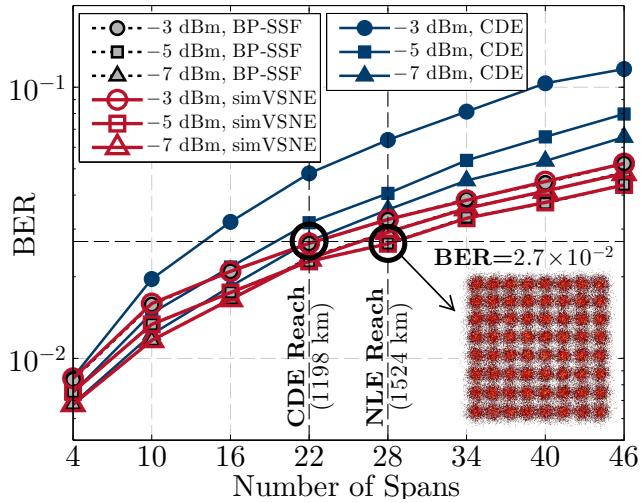
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- The simVSNE is able to match the BP-SSF maximum performance (1 span per step);
- Maximum reach is increased from 1198 km to 1524 km (27% increase);
- Optimum power is increased from -7 dBm to -5 dBm (2 dB increase in nonlinear tolerance).

# simVSNE - Computational Effort

- Computational effort comparison between BP-SSF, symVSNE and simVSNE:

$N_{\text{steps}}$	$N$	<b>SSF</b>	symVSNE		simVSNE	
		$N_{\text{CMs}}$	$N_k$	$N_{\text{CMs}}$	$N_k$	$N_{\text{CMs}}$
28	64	523	4	6113	4	772
<b>14</b>	<b>64</b>	<b>266</b>	4	3063	4	392
7	64	--	5	1794	5	238
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- The SSF method requires a minimum of **14 steps**, yielding **266 CMs**;
- The symVSNE can be applied with only **1 step**, but incurs higher complexity;
- The simVSNE provides an **optimized balance between latency and complexity**:
  - \* **4 steps** and **213 CMs**.

# Time Domain VSNE

## 5.1) Analytical Description

# Time Domain VSNE - Analytical Formulation

- Frequency-domain iXPM filter - `simVSNE[0]`:

$$\tilde{A}_{x/y}^{\text{iXPM}}(\omega_n) = \kappa \left[ \tilde{A}_{x/y}^{\text{CD}}(\omega_n) \left( 2 \sum_{k=1}^N |\tilde{A}_{x/y}(\omega_k)|^2 + \sum_{k=1}^N |\tilde{A}_{y/x}(\omega_k)|^2 - \tilde{\chi}(\omega_n) \right) + \tilde{A}_{y/x}^{\text{CD}}(\omega_n) \sum_{k=1}^N \tilde{A}_{x/y}(\omega_k) \tilde{A}_{y/x}^*(\omega_k) \right],$$

# Time Domain VSNE - Analytical Formulation

- Frequency-domain iXPM filter - **simVSNE[0]**:

$$\tilde{A}_{x/y}^{\text{iXPM}}(\omega_n) = \kappa \left[ \tilde{A}_{x/y}^{\text{CD}}(\omega_n) \left( 2 \sum_{k=1}^N |\tilde{A}_{x/y}(\omega_k)|^2 + \sum_{k=1}^N |\tilde{A}_{y/x}(\omega_k)|^2 - \tilde{\chi}(\omega_n) \right) + \tilde{A}_{y/x}^{\text{CD}}(\omega_n) \sum_{k=1}^N \tilde{A}_{x/y}(\omega_k) \tilde{A}_{y/x}^*(\omega_k) \right],$$

- Applying an inverse Fourier transform, we obtain a **time-domain equivalent**:

$$A_{x/y}^{\text{iXPM}}(t_n) = \kappa \left[ A_{x/y}^{\text{CD}}(t_n) \left( 2P_{x/y}(t_n) + P_{y/x}(t_n) \right) + A_{y/x}^{\text{CD}}(t_n) P_{xy/yx}(t_n) - \chi_{x/y}(t_n) \right],$$

$$P_{x/y}(t_n) = \frac{1}{N_{\text{NLE}}} \sum_{k \in K} |A_{x/y}(t_k)|^2, \quad P_{xy}(t_n) = \frac{1}{N_{\text{NLE}}} \sum_{k \in K} A_x(t_k) A_y^*(t_k), \quad P_{yx}(t_n) = P_{xy}^*(t_n),$$

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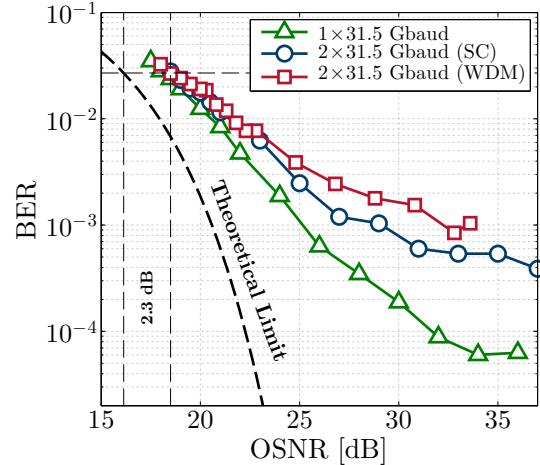
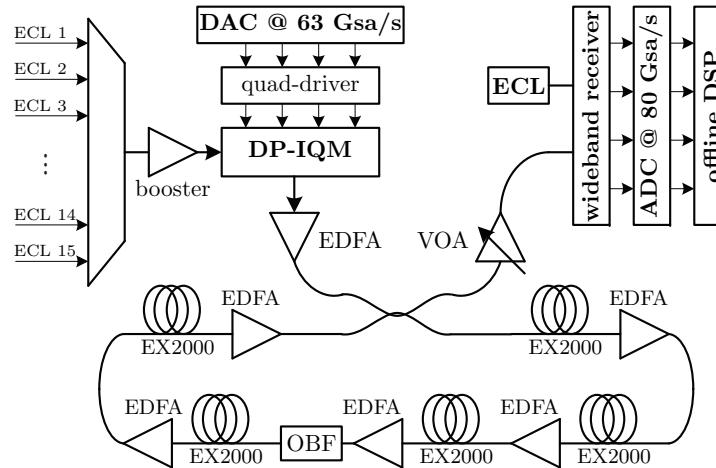
- An identical procedure can be done for the remaining  $\text{simVSNE}\{K\}$  filters, yielding the **weighted VSNE (W-VSNE) filter array**.

# Time Domain VSNE

## 5.2) Experimental Results

# Weighted VSNE - Laboratorial Setup

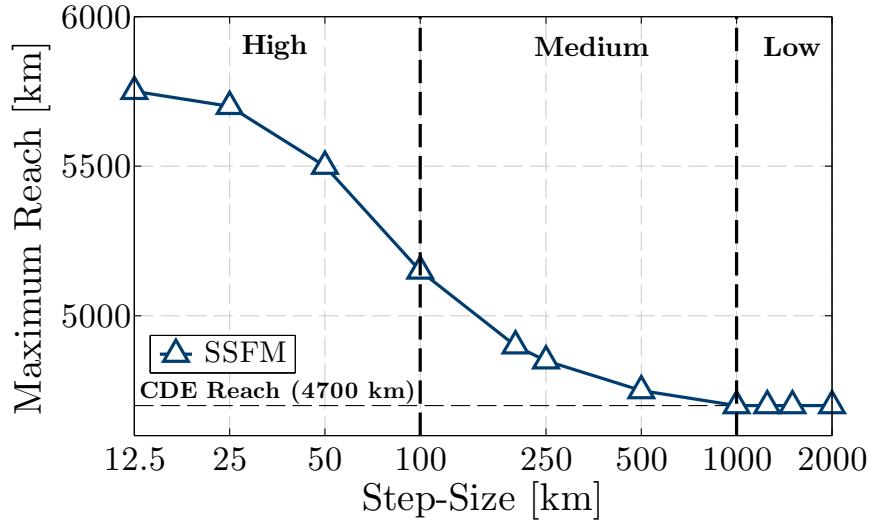
- Laboratorial setup of a **ultra-long-haul 400G PM-16QAM transmission system** implemented in collaborations CPqD, Campinas, Brazil:



- 2×32 Gbaud PM-16QAM** signal;
- Propagation over a recirculating loop composed of **5×ULA (112 μm<sup>2</sup>) spans** with 50 km each;
- 5 WDM superchannels** spaced by **75 GHz**;
- Digital Nyquist pulse shaping.

# Weighted VSNE - Experimental Results

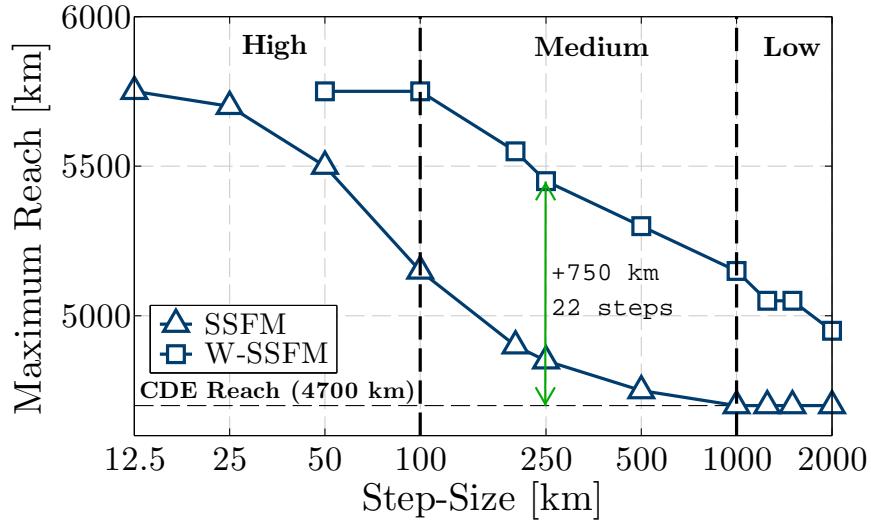
- Estimated maximum reach as a function of the DBP step-size:



- The standard SSFM is **not adequate** for low-complexity DBP.

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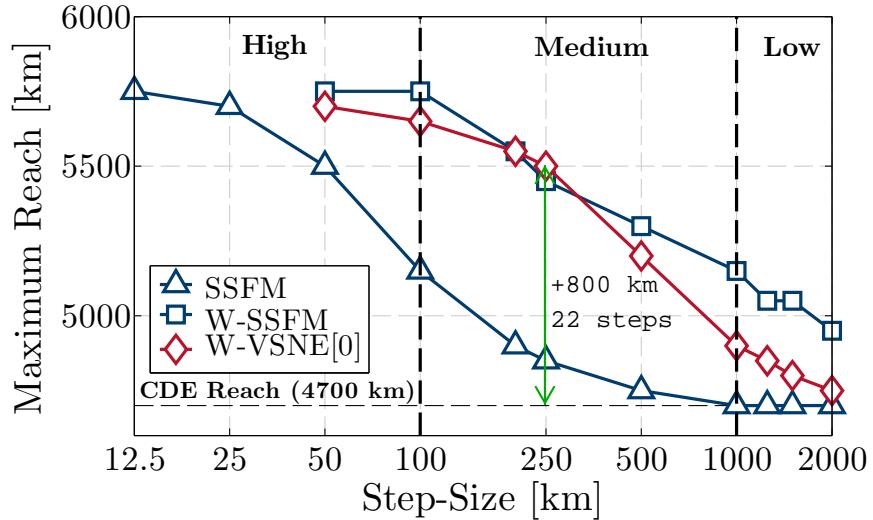
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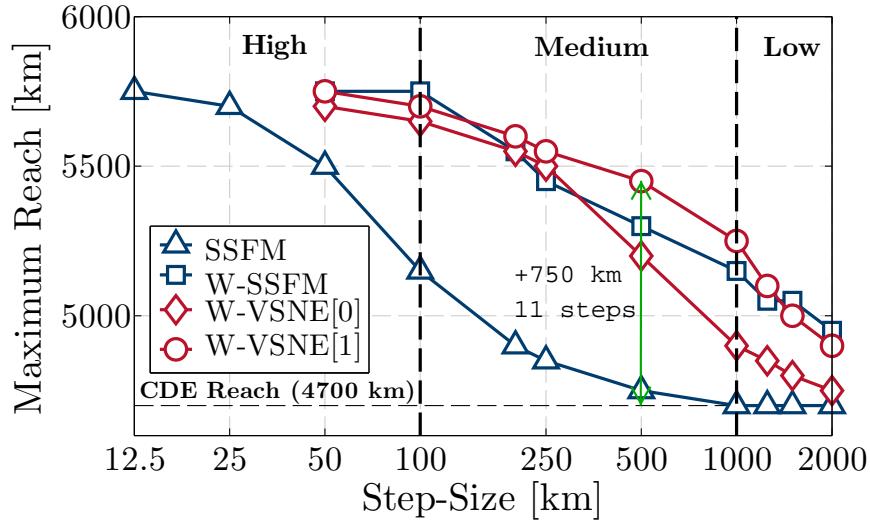
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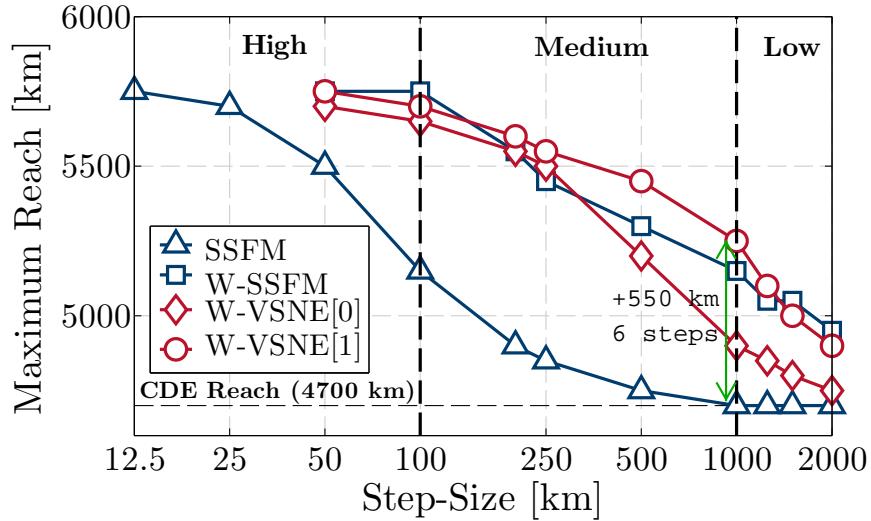
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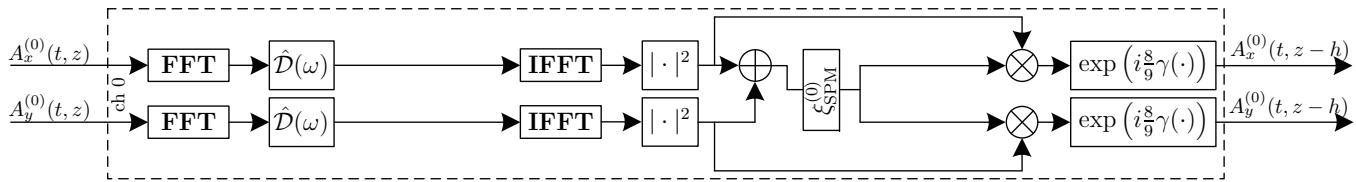
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- The W-SSFMM shows a significantly **increased tolerance** to large step-sizes;
- The **W-VSNE[1]** outperforms the W-SSFMM in the medium performance region;
- Even with a step-size of 1000 km (**6 steps in total**) the W-VSNE[1] enables to extend the signal reach by **550 km**.

# **Multi-Carrier Digital Backpropagation**

## **6.1) Numerical Implementation**

# Coupled Equations DBP - Numerical Implementation

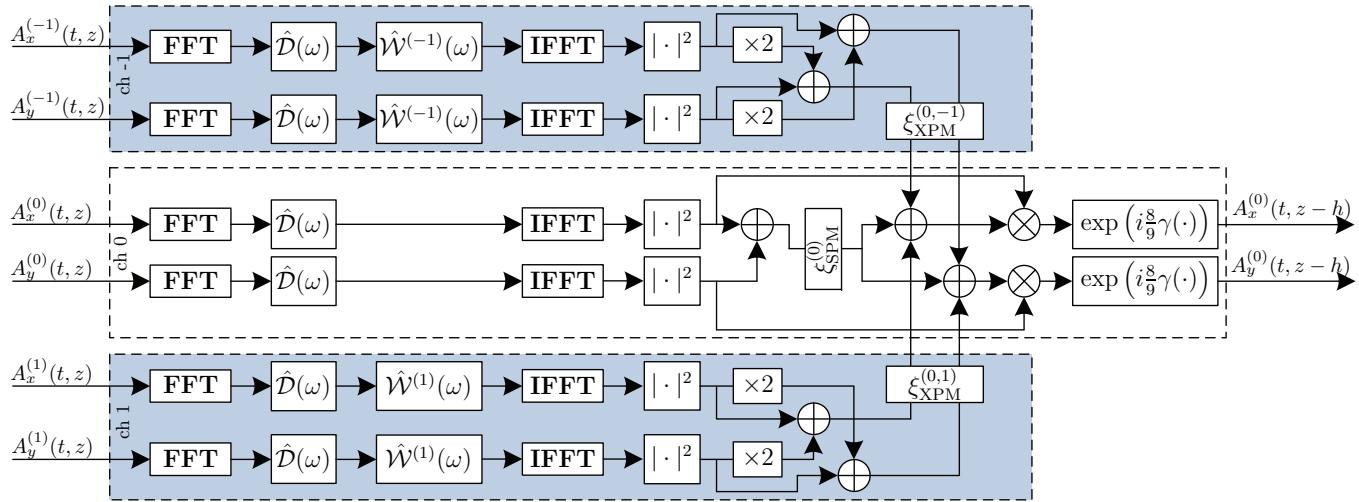
- Numerical implementation of the CE-DBP approach using the asymmetric SSFM:



- SPM** is compensated by the standard SSFM over each subcarrier;

# Coupled Equations DBP - Numerical Implementation

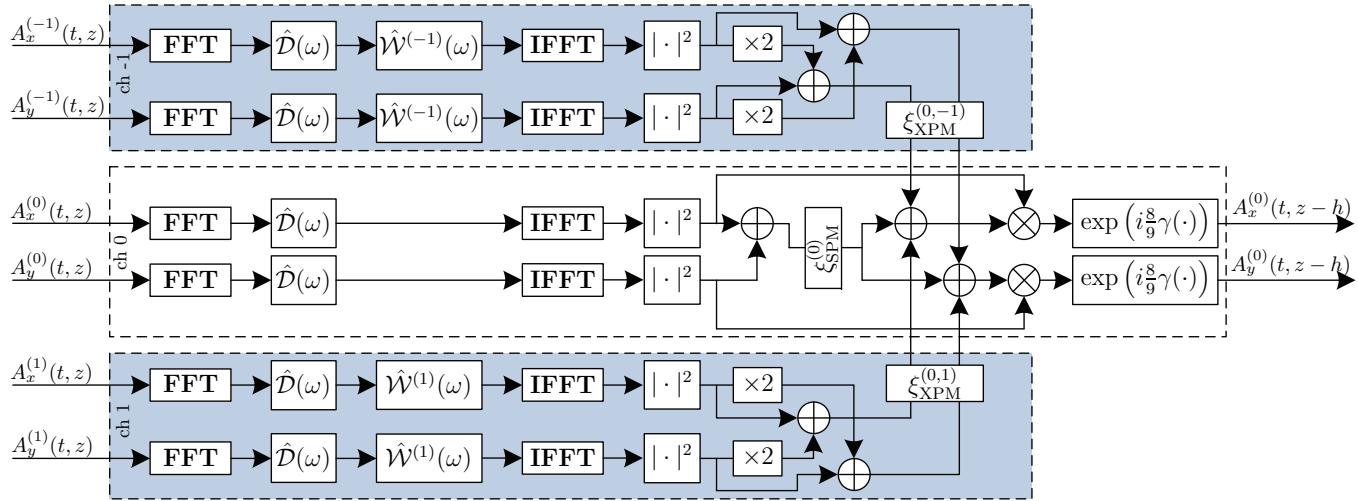
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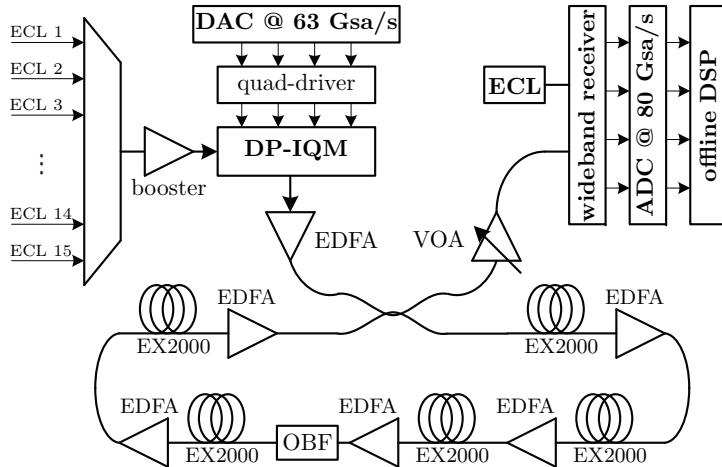
- SPM is compensated by the standard SSFM over each subcarrier;
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  - \* A new operator,  $\hat{W}^{(m)}(\omega)$ , is responsible for the walk-off effect between subcarriers.
- Complexity is similar to that of intra-channel compensation of each subcarrier.

# **Multi-Carrier Digital Backpropagation**

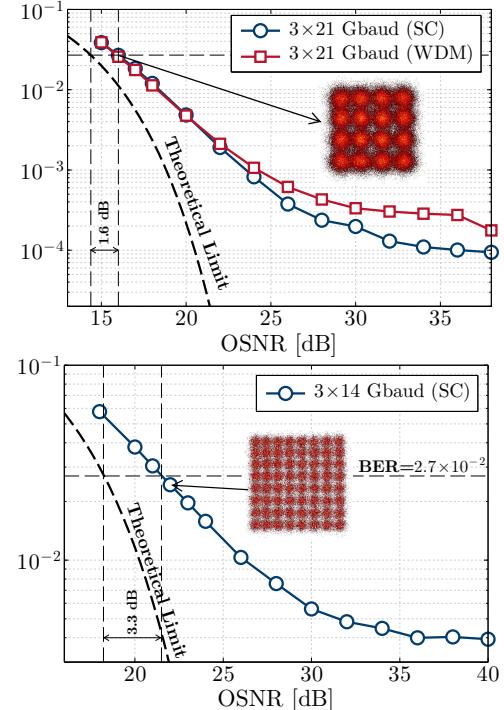
## **6.2) Experimental Results**

# Multi-Carrier DBP - Laboratorial Setup

- Laboratorial setup for **metro** and **ultra-long-haul** 400G PM-16QAM transmission system implemented in collaborations CPqD, Campinas, Brazil:

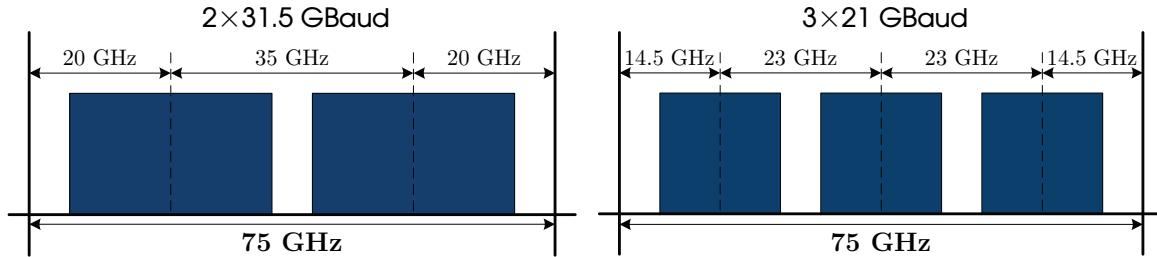


- ULH - **2×32 Gbaud** and **3×21 Gbaud PM-16QAM** signal;
  - \* 5 WDM superchannels in a 75 GHz slot - 5.33 b/s/Hz;
- Metro - **3×14 Gbaud PM-64QAM** signal;
  - \* 1 superchannel in a 50 GHz slot - 8 b/s/Hz;



# 400G Superchannel Configurations

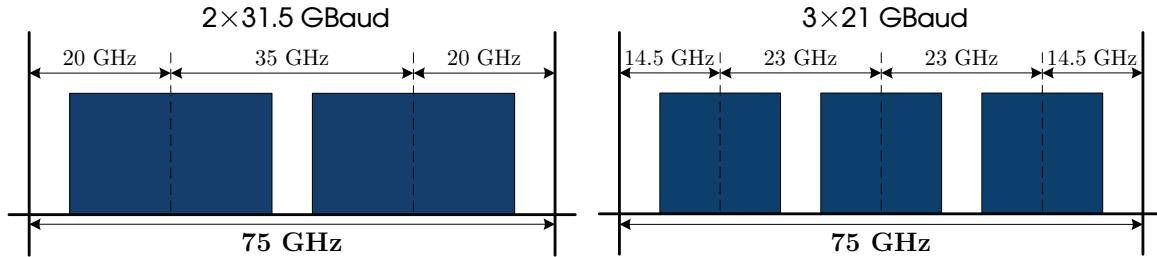
- Superchannel configurations for PM-16QAM:



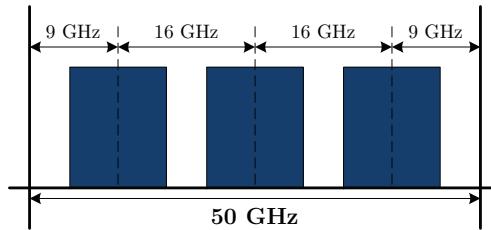
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- Designed for **long-haul and ultra-long-haul applications**.

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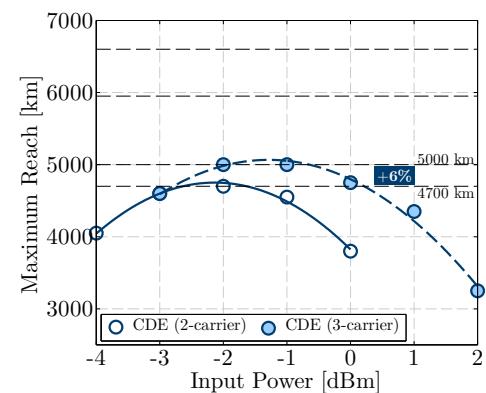
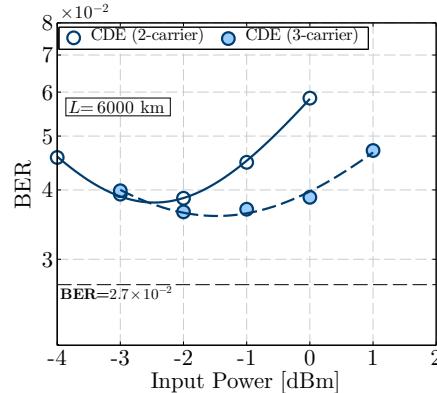
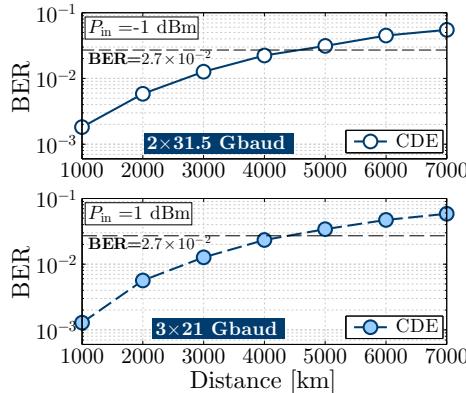
- Net spectral efficiency of 5.33 b/s/Hz;
- Designed for **long-haul and ultra-long-haul applications**.
- Superchannel configuration for PM-64QAM -  $3 \times 14$  GBaud:



- Net spectral efficiency of **8 b/s/Hz**;
- Designed for **metro and long-haul applications** with high spectral efficiency.

# DBP Impact on ULH 400G Performance

- Ultra-long-haul 400G propagation performance:

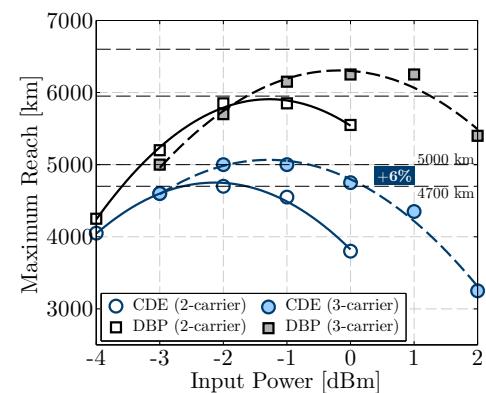
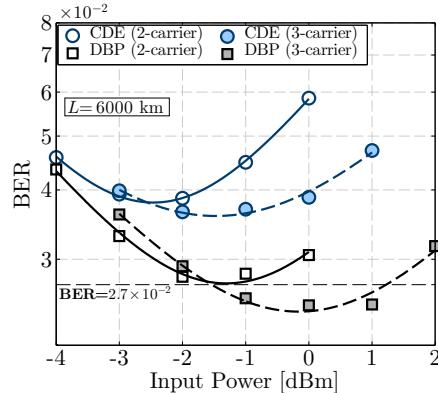
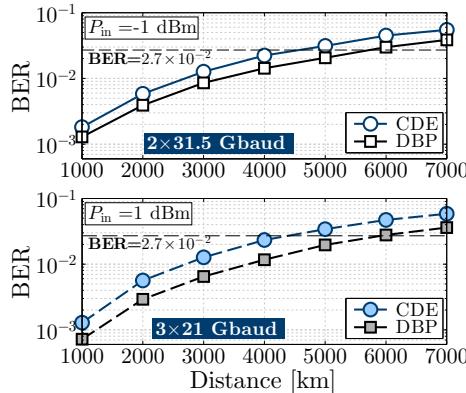


- after CDE, triple-carrier 400G provides an increased reach of approximately **6%**:

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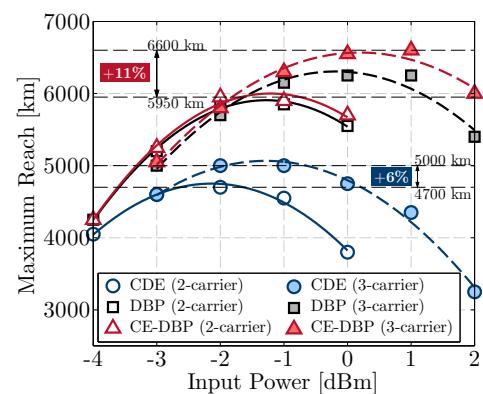
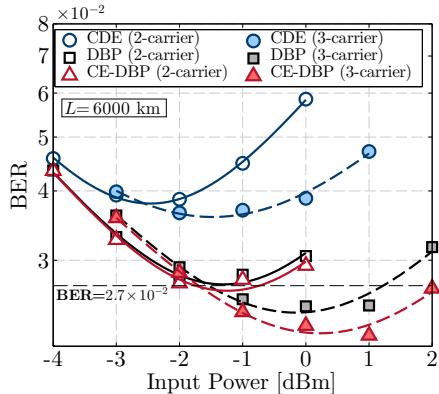
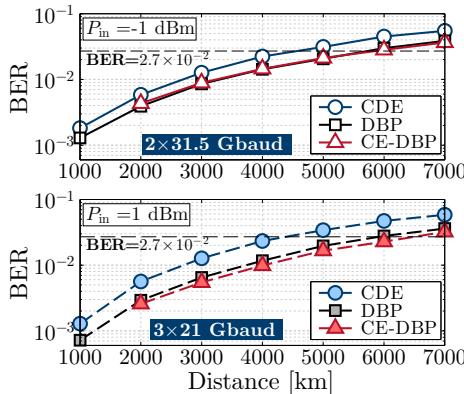


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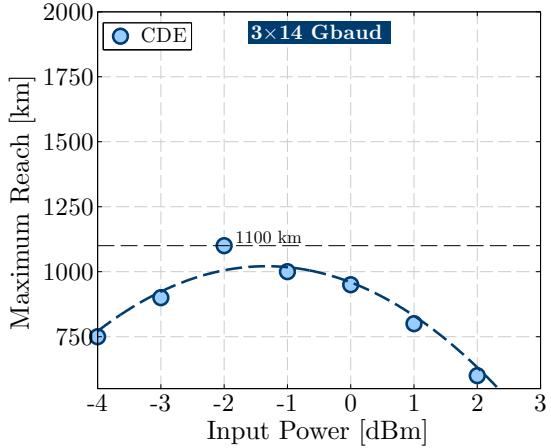
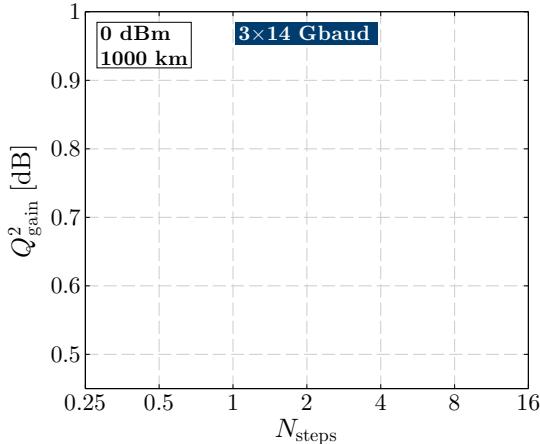
- Ultra-long-haul 400G propagation performance:



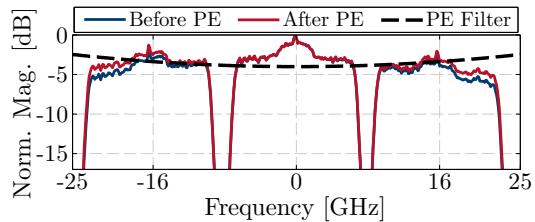
- after CDE, triple-carrier 400G provides an increased reach of approximately **6%**:
  - good agreement with previous works based on simulation and EGN model predictions;
- after CE-DBP, the reach improvement provided by the **triple-carrier 400G** is now of **11%**:
  - CE-DBP performance improves with increasing number of backpropagated subcarriers.
- The overall reach enhancement over CDE is of **1250 km (26%)** and **1600 km (32%)** for the **dual- and triple-carrier 400G** superchannels, respectively.

# MC-DBP: Experimental Results for PM-64QAM

- DBP optimization and performance of 400G superchannels based on PM-64QAM:

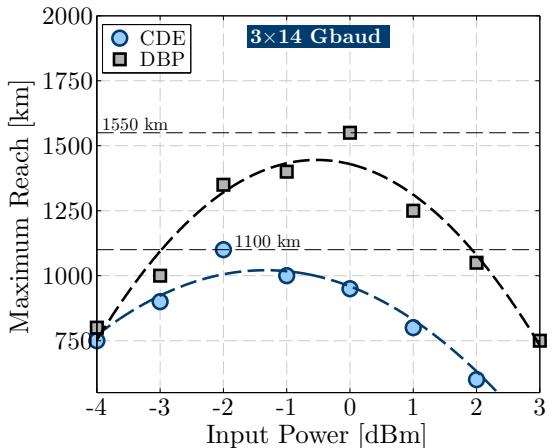
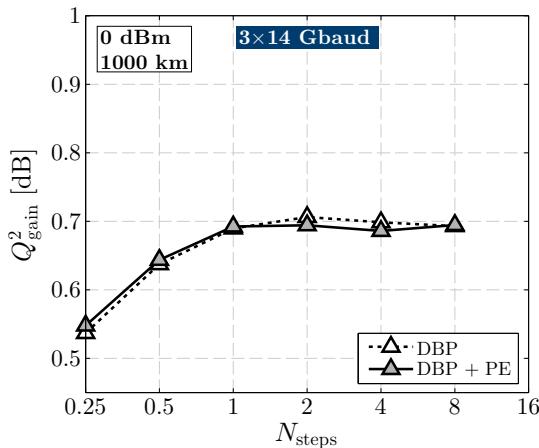


- maximum reach **after CDE** - 1100 km;

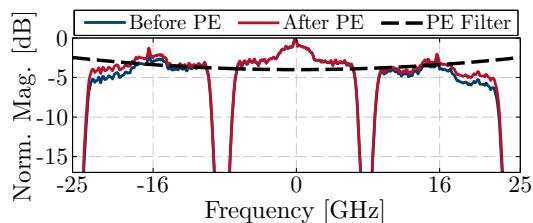


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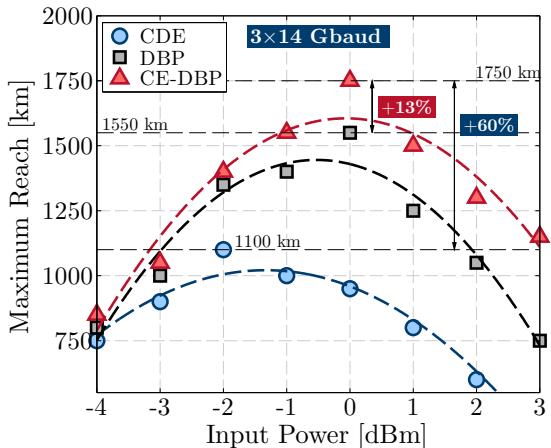
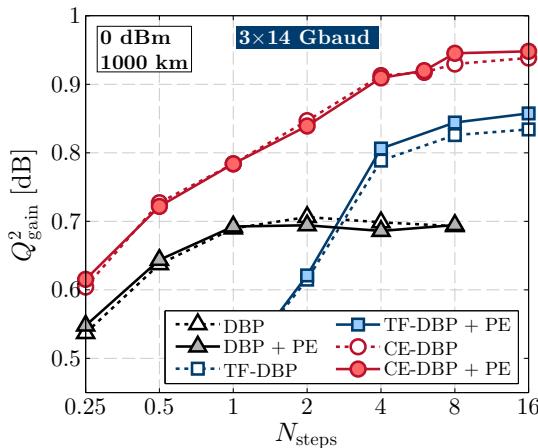


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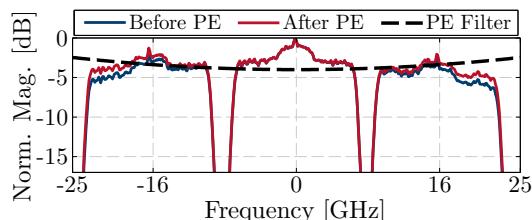


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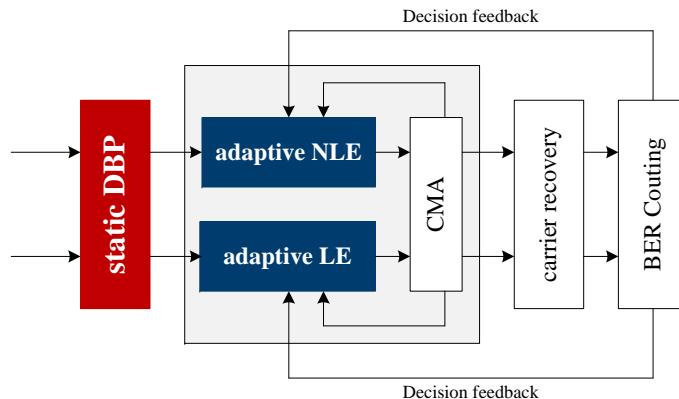
- maximum reach **after CDE** - 1100 km;
- maximum reach **after DBP** - 1550 km;
- maximum reach **after CE-DBP** - 1750 km;
  - 13% over DBP and 60% over CDE.
- The required DBP spatial resolution is very **similar** to that found for ULH based on PM-16QAM.



# **Open Research Topics on Nonlinear Equalization**

# Static + Adaptive Nonlinear Equalization

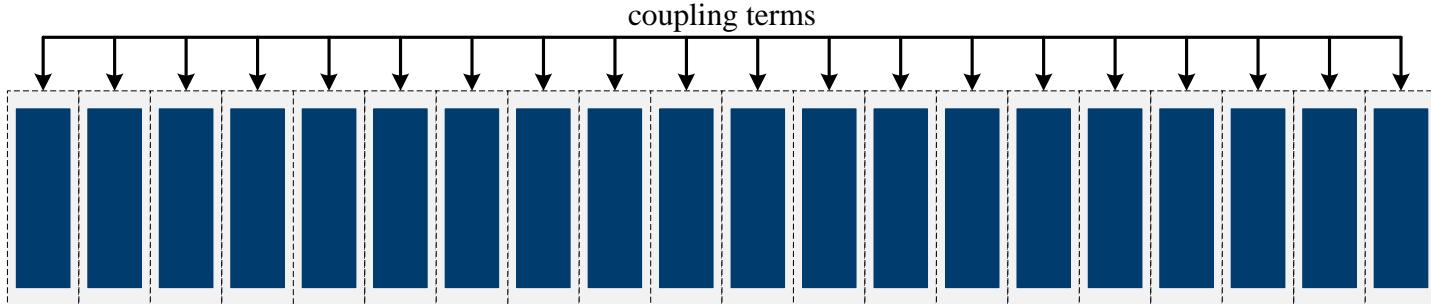
- A practical implementation of DBP requires some **adaptation capabilities**:
  - Uncertainties on the link parameters;
  - Non-homogenous power/gain profile;
  - Other temporal variations due to temperature, bending...
- Fully adaptable DBP would be **too complex** and **very difficult to converge**;
  - The same happens with linear equalization (static + adaptive);



- The bulk estimated nonlinearities can be compensated with a **static equalizer**;
- A low complexity **adaptive equalizer** (Volterra?) optimizes the initial solution.

# DBP for Subcarrier Multiplexing

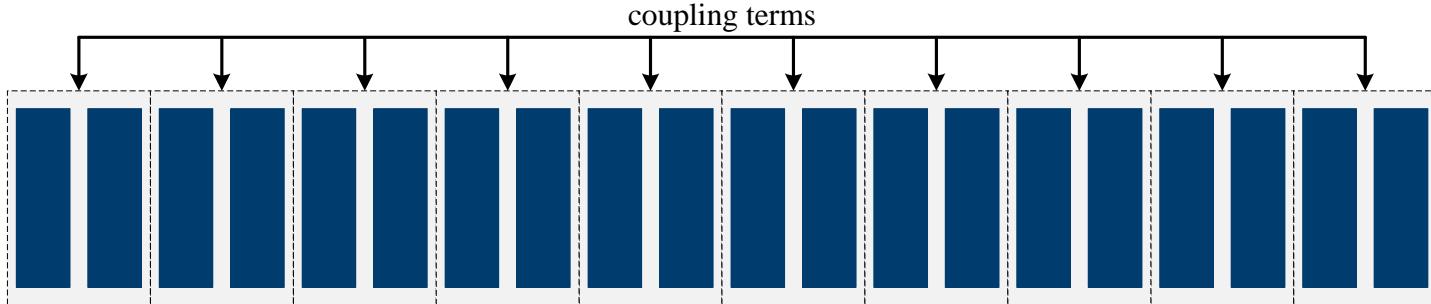
- What is the **best performance vs complexity** trade-off?
  - Low symbol-rate per subcarrier  $\Rightarrow$  **reduced DBP complexity** per subcarrier;
  - Many subcarriers  $\Rightarrow$  **many DBP processing chains**;
  - How to **efficiently deal with FWM?**



- **Too many coupled equations** may be suboptimum;
  - Too many DBP chains - lots of **FFT/IFFT pairs**;
  - A **fully frequency domain** approach can be beneficial (VSNE?).

# DBP for Subcarrier Multiplexing

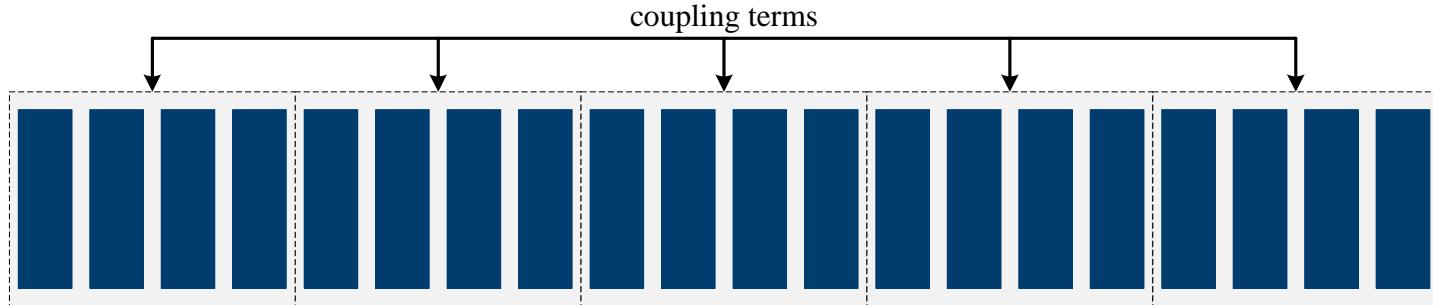
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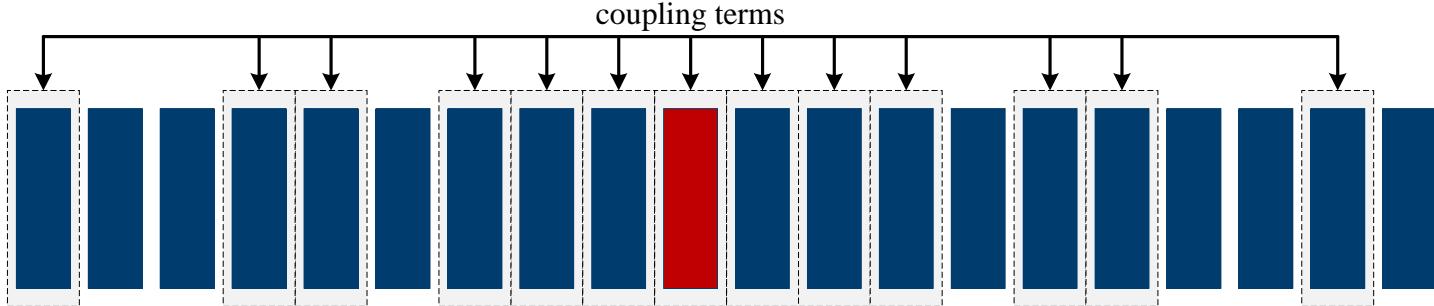
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- The most efficient solution is likely to be based on an **hybrid CE/TF-DBP approach**;
  - Tradeoff between **complexity per DBP chain** and **number of DBP chains**;
- **FWM will be dominant:** Not all subcarriers need to be taken into account.

# The FLEX-ON Project

# What is Flex-ON?

- **Flexible Optical Networks** – Time Domain Hybrid QAM: DSP and Physical Layer Modelling;
- **H2020 project:** Marie Skłodowska-Curie Individual Fellowship;

*“The Marie Skłodowska-Curie actions, named after the double Nobel Prize winning Polish-French scientist famed for her work on radioactivity, support researchers at all stages of their careers, irrespective of nationality. Researchers working across all disciplines, from life-saving healthcare to ‘blue-sky’ science, are eligible for funding. The MSCA also support industrial doctorates, combining academic research study with work in companies, and other innovative training that enhances employability and career development”.*

- **24 months:** October 2015 - October 2017;
- Includes a **secondment** (3 months) with CISCO Photonics.
- Divided into 5 WPs:
  - **WP1** – Digital modulation techniques;
  - **WP2** – Simulation tools and DSP subsystems;
  - **WP3** – NL prediction and equalization tools;
  - **WP4** – Laboratorial implementation, test and validation;
  - **WP5** – Management and dissemination;

# What are the Main Objectives?

- **Scientific Objectives:**

- Optimization of **digital modulation techniques** for spectrally efficient transmission with **high bit-rate granularity**, low energy consumption and robust signal propagation;
- Development of **efficient DSP subsystems** for flexible optical transceivers;
- Development of **NL prediction and equalization tools**;
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## ● Dissemination and Public Engagement:

- Participate in the **European Researchers' Night**;
- Organize an **MSCA seminar** to promote the Marie Skłodowska-Curie Actions among undergraduate students and early stage researchers;
- Design a **website** with both technical and simplified materials;
- Dissemination of optics and photonics at **high-schools**;
- Organize an **yearly workshop** on flexible optical transceivers in POLITO;
- Organize a **workshop at an international conference**.

# Thanks for your attention!

[fernando.guiomar@polito.it](mailto:fernando.guiomar@polito.it)